CONTENTS

1 INTRODUCTION

Engineering economics is the application of economic techniques for the evaluation of design and engineering alternative. The role of engineering economic is to

- Access the appropriateness of a given project.
- Estimate its value
- Justify it from an engineering point of view

It involves the systematic and technical evaluation of analysis, with emphasis on the economic aspects and has the objective of assisting in decision making.

Engineering economics deals with the methods that enable one to take economic decisions towards minimizing costs and/or maximizing benefits to business organizations.

Some Economics Terminology

1. **Annuity:** An amount of money payable to a beneficiary at regular intervals for a prescribed period of time out of a fund reserved for that purpose.
2. **Break Even Point**
   - In business operation, the rate of operations output or sales at which income is equal to operation cost.
   - The percentage of capacity operation of a manufacturing plant at which income will just cover expenses.
3. **Capital:** The non-human ingredients that contribute to the production of goods and services, including land, raw and semi finished materials, tools building machinery and inventories.
4. **Sunk Cost:** A sunk cost is a cost that an entity has incurred and which it can no longer recover by any means. Sunk cost should not be considered when making the decision to continue investing in an

ongoing project, since we cannot recover the cost sunk cost includes marketing study, research & development, training, hiring bonus.

5. **Opportunity Cost:** The cost of an alternative that must be forgone in order to pursue a certain action. It can also be defined as, the value of benefits sacrificed in selecting a course of action among alternatives.
6. **Assets:** An economic resource of entity (including money resources, physical researches, and intangible resources).
7. **Marginal cost:** The cost associated with one additional unit of production, also called the incremental cost.

Types of Business Organization

As an engineer, we should understand the nature of the business organization with which we are associated. This section will present some basic information about the type of organization you should choose should you decide to go into business for yourself.

The three legal forms of business, each having certain advantages and disadvantages, are proprietorships, partnerships, and corporations.

1. **Proprietorships:** A **proprietorship** is a business owned by one individual. This person is responsible for the firm's policies, owns all its assets, and is personally liable for its debts. A proprietorship has two major advantages. First, it can be formed easily and inexpensively. No legal and organizational requirements are associated with setting up a proprietorship, and organizational costs are therefore virtually nil. Second, the earnings of a proprietorship are taxed at the owner's personal tax rate, which may be lower than the rate at which corporate income is taxed. Apart from personal liability considerations, the major disadvantage of a proprietorship is that it cannot issue stocks and bonds, making it difficult to raise capital for any business expansion.
2. **Partnerships:** A **partnership** is similar to a proprietorship, except that it has more than one owner. Most partnerships are established by a written contract between the partners. The contract normally specifies salaries, contributions to capital, and the distribution of profits and losses. A partnership has many advantages, among which are its low cost and ease of formation. Because more than one person makes contributions, a partnership typically has a larger amount of capital available for business use. Since the personal

assets of all the partners stand behind the business, a partnership can borrow money more easily from a bank. Each partner pays only personal income tax on his or her share of a partnership's taxable income.

On the negative side, under partnership law each partner is liable for a business's debts. This means that the partners must risk all their personal assets—even those not invested in the business. And while each partner is responsible for his or her portion of the debts in the event of bankruptcy, if any partners cannot meet their pro rata claims, the remaining partners must take over the unresolved claims. Finally, a partnership has a limited life, insofar as it must be dissolved and reorganized if one of the partners quits.

3. **Corporations** : A corporation is a legal entity created under provincial or federal law. It is separate from its owners and managers. This separation gives the corporation four major advantages:
   - It can raise capital from a large number of investors by issuing stocks and bonds;
   - It permits easy transfer of ownership interest by trading shares of stock;
   - It allows limited liability—personal liability is limited to the amount of the individual's investment in the business; and
   - It is taxed differently than proprietorships and partnerships, and under certain conditions, the tax laws favor corporations. On the negative side, it is expensive to establish a corporation. Furthermore, a corporation is subject to numerous governmental requirements and regulations.

1.1 Origin of Engineering Economy

- The perspective that ultimate economy is a concern to the engineer and the availability of sound techniques to address this concern differentiates this aspect of modern engineering practice from that of the past.
- Pioneer: Arthur M. Wellington, civil engineer in latter part of nineteenth century. He addressed role of economic analysis in the engineering project. The main area of interest for him was railroad building, followed by other contributions which emphasized

techniques depending on financial and actuarial mathematics.

- Later his work was followed by other Eugene Grant who published the first edition of his book which was the milestone in the development of engineering economy.
- In 1942 Woods and Degarmo published the first edition of the book, later entitled engineering economy.

1.2 Principles of Engineering Economics

The four fundamental principles that must be applied in all engineering economic decisions are

(1) The time value of money

(2) Differential (incremental) cost and revenue,

(3) Marginal cost and revenue, and

(4) The trade-off between risk and reward.

Principle 1: A nearby penny is worth a distant dollar. A fundamental concept in engineering economics is that money has a time value associated with it. Because we can earn interest on money received today, it is better to receive money earlier than later. This concept will be the basic foundation for all engineering project evaluation.

Principle 2: All that counts are the differences among alternatives. An economic decision should be based on the *differences* among the alternatives considered. All that is common is irrelevant to the decision. Certainly, any economic decision is no better than the alternatives being considered. Thus, an economic decision should be based on the objective of making the best use of limited resources. Whenever a choice is made, something is given up. The opportunity cost of a choice is the value of the best alternative given up.

Principle 3: Marginal revenue must exceed marginal cost. Effective decision making requires comparing the additional costs of alternatives with the additional benefits. Each decision alternative must be justified on its own economic merits before being compared with other alternatives. Any increased economic activity must be justified on the basis of the fundamental economic principle that marginal revenue must exceed marginal cost. Here, *marginal revenue* means the additional revenue made possible by increasing the activity by one unit (or small unit). *Marginal cost* has an analogous definition. Productive resources—the natural resources, human resources, and capital goods available to make goods and services—are limited.

Therefore, people cannot have all the goods and services they want; as a result, they must choose some things and give up others.

Principle 4: Additional risk is not taken without the expected additional return. For delaying consumption, investors demand a minimum return that must be greater than the anticipated rate of inflation or any perceived risk. If they didn't receive enough to compensate for anticipated inflation and the perceived investment risk, investors would purchase whatever goods they desired ahead of time or invest in assets that would provide a sufficient return to compensate for any loss from inflation or potential risk.

The Seven Principles of engineering economics are:

Principle1: Develop the Alternatives - There must be more than one alternatives, the choice is made among these after subsequent analysis.

Principle 2: Focus on the difference - Only the differences in expected outcomes among the alternatives are relevant to their comparison & should be considered in decision.

Principle 3: Use a consistent viewpoint - The likely outcomes of the various alternatives, initial cost, future saving etc. should be consistently developed from a defined view point.

Principle 4: Use a common unit of measure - Using a common unit of measurement to specify as many of the prospective outcomes as possible will make easier the analysis & comparison of the alternatives.

Principle 5: Consider all relevant criteria - Selection of best alternative required consideration of almost all relevant criteria. This includes both the outcomes specified in the monetary unit & those expressed in some other unit of measurement.

Principle 6: Make uncertainty explicit - Uncertainty is inbuilt in projecting the future outcomes of the alternatives and it should be honored in their analysis and comparison.

Principle 7: Revisit the decision - The initial projected outcomes of the selected alternative should be subsequently compared with the actual results achieved. Revisiting of decision ensure the good result of the final decision.

1.3 Role of Engineer's in Decision Making

What role do engineers play within a firm? What specific tasks are assigned to the engineering staff, and what tools and techniques are available to it to improve a firm's profits? Engineers are called upon to participate in a variety of decisions, ranging from manufacturing, through marketing, to financing decisions. We will restrict our focus, however, to various economic decisions related to engineering projects. We refer to these decisions as **engineering economic decisions.**

In manufacturing, engineering is involved in every detail of a product's production, from conceptual design to shipping. In fact, engineering decisions account for the majority (some say 85%) of product costs. Engineers must consider the effective use of capital assets such as buildings and machinery. One of the engineer's primary tasks is to plan for the acquisition of equipment **(capital expenditure)** that will enable the firm to design and produce products economically.

With the purchase of any fixed asset—equipment, for instance—we need to estimate the profits (more precisely, cash flows) that the asset will generate during its period of service. In other words, we have to make capital expenditure decisions based on predictions about the future. Suppose, for example, you are considering the purchase of a deburring machine to meet the anticipated demand for hubs and sleeves used in the production of gear couplings. You expect the machine to last 10 years. This decision thus involves an implicit 10-year sales forecast for the gear couplings, which means that a long waiting period will be required before you will know whether the purchase was justified.

An inaccurate estimate of the need for assets can have serious consequences. If you invest too much in assets, you incur unnecessarily heavy expenses. Spending too little on fixed assets, however, is also harmful, for then the firm's equipment may be too obsolete to produce products competitively, and without an adequate capacity, you may lose a portion of your market share to rival firms. Regaining lost customers involves heavy marketing expenses and may even require price reductions or product improvements, both of which are costly.

In summary, decision making is always challenging. Analysis of each alternatives by considering all relevant criteria, technically, economically (initial cost and future saving) is must necessary. The role of engineers in the decision making is that evaluate the condition, feasibility and appropriateness of an alternatives of a given project. Estimates its value and justify it from an engineering stand point and finally discover the best alternative for implementation.

Role of engineers in decision making (stepwise duty)

- Understand the problem and define the objectives

- Collect relevant information
- Define the feasible alternative solution and make realistic estimates.
- Identify the criteria for decision making using one or more attribute.
- Evaluate each alternative, using sensitivity analysis to enhance the evaluation
- Select the best alternatives
- Implement the solution and monitor the result.

1.4 Cash Flow

Cash flow is the statement which shows inflows and outflows of cash and cash equivalents during life of project. i.e. actual rupees coming into a firm or actual rupees going out from firm in different time periods. It is the basis for the evaluation of different alternatives.

Cash flow diagram (CFD):

The graphical representation of the cash flows streams i.e. both cash outflows and cash inflows with respect to a time scale is generally referred as cash flow diagram.

It should show three things:

i) A time interval divided into an appropriate number of equal periods.

ii) All cash out flows in each period.

iii) All cash inflows for each period.

[Note: Unless otherwise indicated, all such cash flows are considered to occur at the end of their respective periods.]

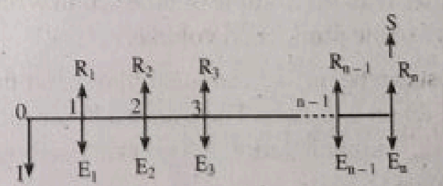

Fig. Cash flow diagram

Where, I = Initial investment

R = Revenue (Benefit)

E = Expenses (cost)

S = Salvage value

How to draw cash flow diagram?

Steps of drawing a cash flow diagram:

1. Horizontal line is time scale with progression of time moving from left to right. (Normally starts from 0 to year N) where end of previous year coincide with beginning of next year.
2. Inflows & outflows of money are expressed by upward and downward arrows respectively. Here inflows of cash represent annual benefit (revenue), salvage values etc and outflows represent initial capital investment, annual cost etc.
3. It is better to show two or more cash flows occurring in the same year individually to make clear connection in between problem statement and cash flows in the diagram.

[Note: Always kept in mind, arrow lengths are approximate proportional to the magnitude of their respective cash flows:]

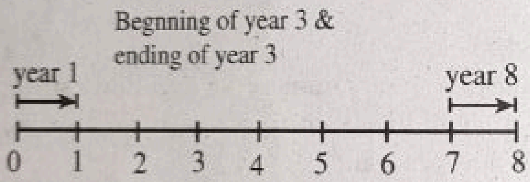

Fig. Typical CFD for 8 years

It shows cash flows begins from year 0 to year N = 8.

1.5 Some Other Useful Topic

1.5.1 Elements of Cost

Cost can be broadly classified into variable cost and overhead cost. Variable cost varies with the volume of production while overhead cost is fixed, irrespective of the production volume.

Variable cost can be further classified into direct material cost, direct labour cost, and direct expenses. The overhead cost can be classified into factory overhead, administration overhead, selling overhead, and distribution overhead.

Direct material costs are those costs of materials that are used to produce the product. Direct labour cost is the amount of wages paid to the direct labour involved in the production activities. Direct expenses are those expenses that vary in relation to the production volume, other than the direct material costs and direct labour costs. Overhead cost is the

aggregate of indirect material costs, indirect labour costs and indirect expenses. Administration overhead includes all the costs that are incurred in administering the business. Selling overhead is the total expense that is incurred in the promotional activities and the expenses relating to sales force. Distribution overhead is the total cost of shipping the items from the factory site to the customer sites.

The selling price of a product is derived as shown below:

(a) Direct material costs+Direct labour costs + Direct expenses = Prime cost

(b) Prime cost + Factory overhead = Factory cost

(c) Factory cost + Office and administrative overhead = Costs of production

(d) Cost of production + Opening finished stock - Closing finished stock = Cost of goods sold

(e) Cost of goods sold + Selling and distribution overhead = Cost of sales

(f) Cost of sales + Profit = Sales

(g) Sales/Quantity sold = Selling price per unit

In the above calculations, if the opening finished stock is equal to the closing finished stock, then the cost of production is equal to the cost of goods sold.

Other Costs/Revenues

The following are the costs/revenues other than the costs which are presented in the previous section:

a. Marginal cost
b. Marginal revenue
c. Sunk cost
d. Opportunity cost

Marginal Cost: Marginal cost of a product is the cost of producing an additional unit of that product. Let the cost of producing 20 units of a product be Rs. 10,000, and the cost of producing 21 units of the same product be Rs. 10,045. Then the marginal cost of producing the 21st unit is Rs. 45.

Marginal Revenue: Marginal revenue of a product is the incremental revenue of selling an additional unit of that product. Let, the revenue of selling 20 units of a product be Rs. 15,000 and the revenue of selling 21 units of the same product be Rs. 15,085. Then, the marginal revenue of selling the 21st unit is Rs. 85.

Sunk Cost: This is known as the past cost of an equipment/asset. Let us assume that an equipment has been purchased for Rs. 1,00,000 about three years back. If it is considered for replacement, then its present value is not Rs. 1,00,000. Instead, its present market value should be taken as the present value of the equipment for further analysis. So, the purchase value of the equipment in the past is known as its sunk cost. The sunk cost should not be considered for any analysis done from now onwards.

Opportunity Cost: In practice, if an alternative (X) is selected from a set of competing alternatives (X,Y), then the corresponding investment in the selected alternative is not available for any other purpose. If the same money is invested in some other alternative (Y), it may fetch some return. Since the money is invested in the selected alternative (X), one has to forego the return from the other alternative (Y). The amount that is foregone by not investing in the other alternative (Y) is known as the opportunity cost of the selected alternative (X). So the opportunity cost of an alternative is the return that will be foregone by not investing the same money in another alternative.

Consider that a person has invested a sum of Rs. 50,000 in shares. Let the expected annual return by this alternative be Rs. 7,500. If the same amount is invested in a fixed deposit, a bank will pay a return of 18%. Then, the corresponding total return per year for the investment in the bank is Rs. 9,000. This return is greater than the return from shares. The foregone excess return of Rs. 1,500 by way of not investing in the bank is the opportunity cost of investing in shares.

1.5.2 Break-Even Analysis

The main objective of break-even analysis is to find the cut-off production volume from where a firm will make profit.

Let s = selling price per unit

v = variable cost per unit

FC = fixed cost per period

Q = volume of production

The total sales revenue (S) of the firm is given by the following formula: $S = s \times Q$

The total cost of the firm for a given production volume is given as

$$TC = \text{Total variable cost} + \text{Fixed cost}$$

$$= v\,Q + FC$$

The linear plots of the above two equations are shown in Fig. below. The intersection point of the total sales revenue line and the total cost line is called the break-even point. The corresponding volume of production on the X-axis is known as the break-even sales quantity. At the intersection point, the total cost is equal to the total revenue. This point is also called the no-loss or no-gain situation. For any production quantity which is less than the break-even quantity, the total cost is more than the total revenue. Hence, the firm will be making loss.

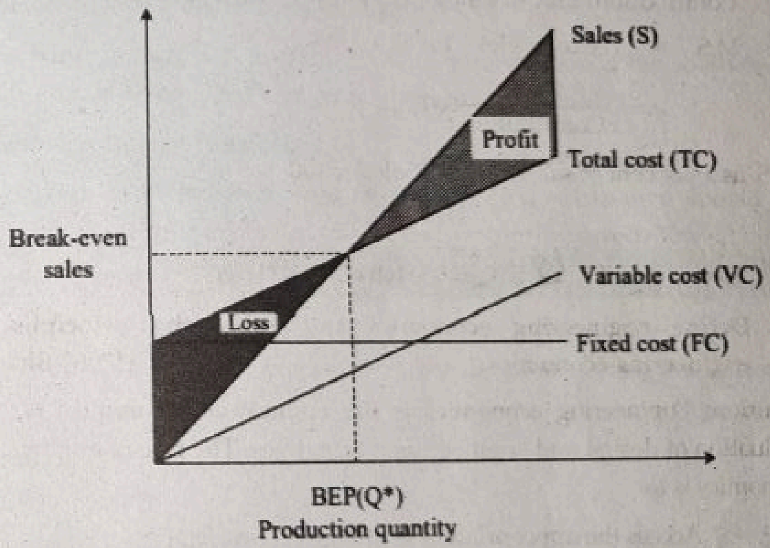

Fig: Break even Chart

For any production quantity which is more than the break-even quantity, the total revenue will be more than the total cost. Hence, the firm will be making profit.

$$\text{Profit} = \text{Sales} - (\text{Fixed cost} + \text{Variable costs})$$

$$= s \times Q - (FC + v \times Q)$$

The formulae to find the break-even quantity and break-even sales quantity

$$\text{Break-even quantity} = \frac{\text{Fixed cost}}{\text{Selling price/unit - Variable cost/unit}}$$

$$= FC/\,(s - v) \text{ (in units)}$$

$$\text{Break-even sales} = \frac{\text{Fixed cost}}{\text{Selling price/unit - Variable cost/unit}} \times \text{Selling price/unit}$$

$$= \frac{FC}{s - v} \times s \quad (\text{Rs.})$$

The contribution is the difference between the sales and the variable costs. The margin of safety (M.S.) is the sales over and above the break-even sales. The formulae to compute these values are Contribution = Sales – Variable costs

Contribution/unit = Selling price/unit – Variable cost/unit

$$\text{M.S.} = \text{Actual sales} - \text{Break-even sales}$$

$$= \frac{\text{Profit}}{\text{Contribution}} \times \text{sales}$$

M.S. as a per cent of sales = (M.S./Sales) x 100

Old Question Solution

1. **Define engineering economics and enlist the principles of engineering economics.** **[2069 Bhadra]**

Solution: Engineering economics is the application techniques for the evaluation of design and engineering alternatives. The role of engineering economics is to

- Access the appropriateness of a given project.
- Estimate its value
- Justify it from an engineering point of view

Principles of Engineering Economics:

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There must be more than one alternatives, the choice is made among these after subsequent analysis.

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Uncertainty is inbuilt in projecting the future outcomes of the alternatives and it should be honored in their analysis and comparison.

Principle 7: Revisit the decision

The initial projected outcomes of the selected alternative should be subsequently compared with the actual results achieved. Revisiting of decision ensure the good result of the final decision.

2. **Explain the roles of engineers in the decision making examples:** **[2070 Magh]**

Solution:

Refer to [1.3]

3. **Scarcity is the emerging issue in engineering field. How the study of economics does help an engineer for decision making?** **[2070 Bhadra]**

Solution:

Scarcity is a naturally occurring limitation on the resources of that cannot be replenished. Scarcity may be classified as:

i) Financial scarcity

ii) Material scarcity

iii) Labour scarcity

iv) Technology scarcity

v) Machine scarcity

vi) Scarcity in time

Scarcity is the emerging issue in the field of engineering. To implement any kind of project it demands a lot of financial resources, material, machine, long time various kinds of manpower. Proper arrangement, management and coordination were all those factors is much necessary for any project to succeed. It is always desirable to complete the project within possible short period, economically without compromising on quality. For this wisely decision making plays vital role & engineers must be responsible for this.

For Role of engineers in Decision making: Refer to (1.3)

4. List out the principles of engineering economics. [2071 Magh]

Solution:

Refer to [1.2]

5. Defined engineering economics. Write down the principles of engineering economic analysis. [2071 Bhadra]

Solution:

Refer to [2069 Bhadra]

6. Explain why the subject of engineering economics is important to civil engineer. [2072 Ashwin]

Solution:

Fields of civil engineering involves construction of big projects with huge investment which directly related to the economy of the society. There occur a lot of alternatives; selection of best alternatives is always a very important task. So, decision making is always challenging and tough. Analysis of each alternatives by considering all relevant criteria, technically, economically (initial cost, future saving) is must necessary. Civil engineers can play vital role in decision making to evaluate the condition, feasibility and appropriateness of an alternatives of a given project, estimate its value and justify it from an engineering stand point & finally discover the best alternative for implementation. For this knowledge about both in the field of civil engineering and engineering economics is necessary which clearly shows the important of engineering economics to civil engineer.

7. Define engineering economics. Write down the principles of engineering economic analysis. [2073 Bhadra]

Solution:

Refer to 2069 Bhadra

8. "Knowledge of engineering economics helps in decision making process". Justify it by the principles of engineering economics. [2073 Magh]

Solution:

Economics models help managers and economists analyze the economics decision-making process. Each model relies on a number of assumptions or basis factors that are present in all decision situations. The fundamental basis of economic decision making is individuals or organization's desire to maximize benefits.

Following are the reasons to have knowledge of engineer's economics for engineers in decision making process.

- Develop the alternatives
- use a common unit of measure
- use a consistent viewpoint
- consider all relevant criteria
- Make uncertainty explicit
- Revisit the decision

Engineering economics is useful to indentify alternatives uses of limited resources and to select the preferred course of action. It is devoted to the problem solving and decision making at the operations level.

9. State and explain principles of engineering economics. [2074 Bhadra]

Solution:

Refer to [1.2]

10. **Define term engineering economics. Explain principles of engineering economy.** **[2075 Bhadra]**

Solution:

Refer to [2069 Bhadra] Q.N. 1

11. **Define engineering economics. Why engineering economics is considered as important aspect for making decision for engineers? Explain.**

[2076 Bhadra]

Solution:

Refer to [2069 Bhadra] Q.N. 1 and Chapter 1.3

2 INTEREST & TIME VALUE OF MONEY

2.1 Introduction to Time Value of Money

The idea that a rupees today worth more than a rupees in the future because the rupees today can earn interest.

So, it is defined as "Time value of money is the time dependent value of money stemming both from changes in purchasing power of money (inflation and deflation) and from the real earning potential of alternative investment overtime." Since money has the ability to earn interest. Its value increase with time. Hence it is the relationship between interest and time.

The value of a currency expressed in terms of the amount of goods or services that one unit of money can buy is called purchasing power. As time lefts purchasing power of money is reduced.

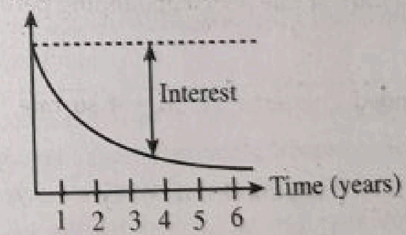


Fig.: Time vs value of money

2.2 Simple Interest

Simple interest is interest earned on only the principal amount during each interest period. In other words, with simple interest, the interest earned during each interest period does not earn additional interest in the remaining periods, even though you don't withdraw it.

In general, for a deposit of P dollars at a simple interest rate of r for N periods, the total earned interest would be, $I = (r \times P) N$

2.3 Compound Interest

When the interest earn in each period is calculated on the basis of the total amount at the end of the previous period & this total amount includes the original principal plus the accumulated interest that has been left in the account is called compound interest.

So, the total amount, at the end of N period when sum P is deposited at interest rate 'i' is $F = P(1+i)^N$

2.3.1 Nominal Interest Rate

The nominal interest rate is periodic interest rate times the number of periods per year. It does not include any consideration of compounding. For example, a nominal annual interest rate of 12% based on monthly compounding means 1% interest rate per month.

If a financial institution uses a unit of time other than a year. A month or quarter (eg: when calculating interest payments), the institution usually quotes the interest rate on annual basis. Commonly this rate is stated as r% compounded M - ly

Where, r = nominal interest rate per year

M = compounding frequency or number of interest periods per year

r/M = interest rate per compounding period

For example,

r = 12 % compounded quarterly, i.e. M = 4 so rate = $\frac{12\%}{4}$ (3% per 3 month)

As well as, r = 12 % compounded monthly.

i.e. M = 12 so rate = $\frac{12\%}{12}$ (1% per month)

2.3.2 Effective Interest Rate

The actual rate of interest earned during one year is known effective rate and it is also expressed on the annual basis unless specifically stated otherwise; Effective interest rate is represented by 'i'

$$\text{Then } i = \left(1+\frac{r}{m}\right)^m - 1$$

Where, m = compounding period per year

Also,

$$i_N = (1+i)^m = 1$$

Where,

N = no. of compounding period

i = interest rate per compounding period.

M = compounding period per year.

Example: From above formula the effective interest rate for 12% Compounded semi - annually,

$$i = \left(1+\frac{0.12}{2}\right)^2 - 1 = 12.36\% \quad [\because r = 12\% \ \& \ m = 2]$$

An interest rate takes two forms: nominal interest rate and effective interest rate. The nominal interest rate does not take into account the compounding period. The effective interest rate does take the compounding period into account and thus is a more accurate measure of interest charges.

A statement that the "interest rate is 10%" means that interest is 10% per year, compounded annually. In this case, the nominal annual interest rate is 10%, and the effective annual interest rate is also 10%. However, if compounding is more frequent than once per year, then the effective interest rate will be greater than 10%. The more often compounding occurs, the higher the effective interest rate.

All of the formulas used in making time value calculations are based on effective interest rates. Therefore, whenever the interest rate that is provided is a nominal rate, it is necessary to convert it to an effective interest rate. As shown below, an effective interest rate, i, can be calculated for any time period longer than the compounding period.

The most common way that nominal interest rates are stated is in the form 'x% per year compounded y' where x = interest rate and y = compounding period. An example is 18% per year compounded monthly. When interest rates are stated this way, the simplest effective rate to get is the one over the compounding period because all that is required is a simple division. For example, from the interest rate of 18% per year compounded monthly, a monthly interest rate of 1.5% is obtained (i.e., 18% per year/12 compounding periods per year) and this is an effective rate because it is the rate per compounding period. To get an effective rate for any period longer than the compounding period use the effective interest rate formula.

$$i = (1+r/m)^m - 1$$

Where: I = effective interest rate per period

r = nominal interest rate per period

m = number of time interest is compounded per period

The types of calculations used to obtain effective interest rates are summarized in Table below

Table: Summary of Calculations Involved in Finding Effective Rates

Interest Statement	To Find i for Compounding Period	To Find i for any Period Longer than Compounding Period
i = 1% per month	i is already expressed over compounding period	Use effective interest rate equation
i = 12% per year compounded quarterly	Divide 12% by 4	Use effective interest rate equation
i = nominal 16% per year compounded semiannually	Divide 16% by 2	Use effective interest rate equation
i = effective 14% per year compounded monthly	Use effective interest rate equation and solve for r/m	For effective i values other than yearly, solve for r in effective interest rate equation and then proceed

2.3.3 Continuous compounding

Continuous compounding can be thought of as making compounding period infinitesimally small. Here cash flows occurs at discrete interval but it is compounded continuously.

Therefore it can be achieved by taking limit of m (no of compounding periods in year) to infinity.

$$\text{So, } i = \lim_{m \to \infty}\left[\left(1+\frac{r}{m}\right)^m - 1\right]$$

$$\therefore \boxed{i = e^r - 1} \quad \ldots\ldots\ldots\ldots\text{(a)}$$

$$\text{or, } e^r = (1 + i)$$

We have,

$$F = P(1 + i)^N$$

$$\text{or, } F = P e^{rN}$$

$$F = P\{F/P, r\%, N\}$$

r % is used to denote the nominal rate and the use of continuous compounding.

2.4 Economic Equivalence

The process of comparing two different cash amounts at different points in time is called economic equivalence. It indicates that different amount of money at different time periods are equivalent by considering the time value of money. Different sums of money at different times are equal in economic value

Calculations for determining the economic effects of one or more cash flows are based on the concept of economic equivalence. Economic equivalence refers to the fact that cash flow whether a single payment or a series of payments can be converted to an equivalent cash flow at any point in time.

The equivalent value of an amount that is borrowed now, at future time period at a given interest rate depends on the type of interest whether simple or compound and the different loan repayment arrangements like payment of accumulated interest annually and principal at the end of the stipulated interest periods or payment of both the principal and interest at the end interest periods or payment of uniform amounts annually that comprises a portion towards the payment of principal amount and remaining for the accumulated interest throughout the interest periods.

For example, we could find the equivalent future value of F or a present amount P at interest rate i at period n; or we could determine the equivalent present value of P of n equal payment A.

Equivalence Calculations

General Principles are:

Principle 1: Equivalence calculations made to compare alternatives

require a common time basis:

To establish an economic equivalence between two cash flow amounts, a common base period must be selected.

Principle 2: Equivalence depends on interest rate:

The equivalence between two cash flows is a function of the magnitude and timing of individual cash flows and the interest rate will destroy the equivalence between these two sums, as we will demonstrate in Example 4

Principle 3: Equivalence calculation may require the conversion of multiple payment cash flows to a single cash flow.

Principle 4: Equivalence is maintained regardless of point of view.

Solved Example

1. **Suppose you deposit \$1000 in a bank savings accounts that pays interest rate of 10% compounded annually. Assume that you don't withdraw the interest earned at the end of each period (one year), but let it accumulate. How much would you have at the end of year 3?**

Solution:

Given: $P = \$1000, N = 3$ years, and $i = 10\%$ per year

Find: F, $F = \$1000\,(1 + 0.10)^3 = \$1.33\%$ $[\because \text{ from } F = P(1 + i)^N]$

2. **In 1626, Peter of the Dutch west company paid \$24 to purchase Manhattan Island in New York from the Chinese. In retrospect, if peter had invested \$24 in a saving account that earned 8% interest rate, how much would it be worth in 2007?**

Solution:

Given $P = \$24, i = 8\%$ per year & $N = 381$ yrs (2007 – 1626)

Find F, based on (a) 8% simple interest, $F = \$24\,[1 + 0.08 \times 381] = \$755.52$

(b) 8% compound interest, $F = \$24\,(1 + 0.08)^{381}$

$= \$130,215,319,909,015$

3. **Suppose you are offered the alternative of receiving either \$ 3,000 at the end of 5 yrs or P dollars today. There is no question that the \$ 3,000 will be paid in full (no risk). Because you have no current need for money, you deposit the p dollars in an account that pays 8% interest. What value of P would make you indifferent to choose between p dollars today and the promise of \$ 3,000 at the end of 5yrs?**

Solution:

Given: $F = \$3,000$ $N = 5$ yrs and $i = 8\%$ per year

Find: P $F = P(1 + i)^N$

$$P = \frac{F}{(1 + i)^N} = \frac{3000}{(1 + 0.08)^5} = \$\,2,042$$

So, it is clear that if P is anything less than \$ 2,042 you would prefer the promise of \$ 3,000 in five years to P dollars today. Otherwise prefer \$ 3,000

4. **In example 2.3, we determined that, given an interest rate of 8% per year, receiving \$ 2,042 today is equivalent to receiving \$ 3,000 in five yrs. Are these cash flows equivalent at an interest rate of 10%?**

Solution:

Given: $P = \$2,042$ $i = 10\%$ & $N = 5$ yrs

Find: F $F = P(1 + i)^N = 2042\,[1 + 0.10]^5 = \$\,3,289$

Since the amount is greater than \$ 3,000, the change in interest rate destroys the equivalence between the two cash flows.

5. **Suppose you have invested Rs. 1000 at present. How long does it take for your investment to double if interest rate is 8% compounded annually?**

Solution: Let investment F will become 2P after N yrs

$$F = 2P$$

$$\text{From } F = P(1 + i)^N$$

$$2P = P(1 + i)^N$$

$$\text{or, } 2 = (1 + 0.08)^N$$

$$1.08^N = 2$$

$$N \log 1.08 = \log 2 \quad [\because \text{ taking log on both sides}]$$

$$\therefore \quad N = 9 \text{ yrs.}$$

6. **Find effective interest rate when nominal rate of interest is 18% per year & compounding is (i) Monthly (ii) Daily (iii) Hourly**

(iv) **Continuously.**

Solution:

(i) Monthly, $i = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.18}{12}\right)^{12} - 1 = 19.561\%$

(ii) Daily, $i = \left(1 + \frac{0.18}{365}\right)^{365} - 1 = 19.716\%$

(iii) Hourly, $i = \left(1 + \frac{0.18}{365 \times 24}\right)^{365 \times 24} - 1 = 19.721\%$

(iv) Continuously, (from equation (a) from 2.33).

$$i = e^r - 1 = e^{0.18} - 1 = 19.7217\%$$

7. **If a lender charges 12% interest, compounded quarterly, what effective annual interest rate is the lender charging?**

Solution:

$$i = [1 + (0.12 / 4)]^4 - 1$$
$$= (1.03)^4 - 1 = .1255 = 12.55\%$$

8. **If a lender charges 12% interest, compounded monthly, what is the effective interest rate per quarter?**

Hint: m = number of compounding periods per quarter

Solution:

Let, i = effective interest rate per quarter.

r = 12% / 4 = 3% and m=3 (No. of months per quarter)

$i = [1 + (0.03 / 3)]^3 - 1 = (1.01)^3 - 1 = 0.0303 = 3.03\%$

9. **If a money lender charges interest 1% per month what is the effective interest rate per year?**

Solution:

i_N = effective interest rate per year

M = 12

$i_N = [1 + 0.01]^{12} - 1 = 0.1268 = 12.68\%$ per year.

2.5 Development of Interest Formula

As we begin to compare series of cash flow instead of single payments the required analysis become more complicated. However,

when patterns in cash flow transactions can be identified, we can take advantage of these patterns by developing concise expression for computing either present or future worth of the series. We will classify the five major categories of cash flow transactions; develop interest formulas for them.

1. **Single Cash flow :** It involves the equivalence of a single present amount and its future worth. So it deals with only a single present amount P and its future worth F and is given by

$$F = P(1 + i)^N$$

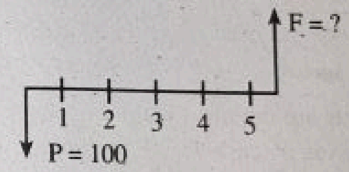


Fig.: Single Cash Flows

2. **Equal payment series:** This describes the cash flow of the common installment loan contract, which arranges repayment of the loan in equal periodic installments.

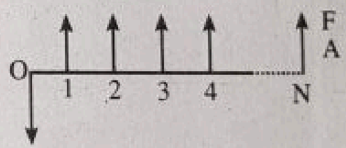


Fig: Uneven Series

$$F = A\frac{[(1 + i)^N - 1]}{i}$$

Where, A = Annual payments or incomes

Payments are made at the end of each equal periods. These transactions are graphically illustrated in figure. Looking at this diagram, we see that if an amount A is invested at the end of each period, for N periods, the total amount F that can be withdrawn at the end of the N periods will be the sum of the compound amounts of the individual deposits.

As shown in figure, the A dollars we put into the fund at the end of the first period will be worth $A(1 + i)^{N+1}$ at the end of N periods. The A dollars we put into the fund at the end of the second period will be worth $A(1 + i)^{N-2}$, and so forth. Finally, the last A dollars that we contribute at the end of the Nth period will be worth exactly A dollars at that time. This means that there exists a series of the form.

$$F = A(1+i)^{N-1} + A(1+i)^{N-2} + \dots + A(1+i) + A$$

or, expressed alternatively,

$$F = A + A(1+i) + A(1+i)^2 + \dots + A(1+i)^{N-1} \quad \dots\dots(1)$$

Multiplying Eq. (1) by (1 + i) results in

$$(1+i)F = A(1+i) + A(1+i)^2 + \dots + A(1+i)^N \quad \dots\dots(2)$$

Subtracting Eq. (1) from Eq. (2) to eliminate common terms gives us

$$F(1+i) - F = -A + A(1+i)^N$$

Solving for F yields,

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A(F/A, i, N)$$

3. **Linear gradient series:**

- The amounts are not always uniform, when many transactions involves series of cash flows but they vary in some regular difference.
- One common pattern of variation occurs when each other cash flow in a series increases (or decreases) by a fixed amount.

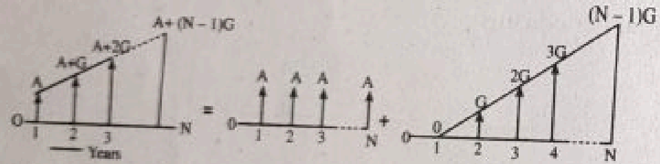


Fig: Linear Gradient Series

Present - Worth Factor: Linear Gradient: Find P, Given G, N, and I

How much would you have to deposit now to withdraw the gradient amounts specified in figure? To find an expression for the present amount P we apply the single - payment present-worth factor to each term of the series and obtain.

$$P = 0 + \frac{G}{(1+i)^2} + \frac{2G}{(1+i)^3} + \dots + \frac{(N-1)}{(1+i)^N}$$

or, $$P = \sum_{n=1}^{N} (n-1)G(1+i)^{-n}$$

Letting G = a and $1/(1+i) = x$ yields

$$P = 0 + ax^2 + 2ax^3 + \dots + (N-1)ax^N$$
$$= ax[0 + x + 2x^2 + \dots + (N-1)x^{N-1}]$$

Since an arithmetic- geometric series $\{0, x, 2x^2, \dots, (N-1)x^{N-1}\}$ has the finite

$$0 + x + 2x^2 + \dots + (N-1)x^{N-1} = \left[\frac{1 - Nx^{N-1} + (N-1)x^N}{(1-x)^2}\right]$$

We can rewrite Eq. as

$$P = ax^2\left[\frac{1 - Nx^{N-1} + (N-1)x^N}{(1-x)^2}\right]$$

Replacing the original values for A and x, we obtain

$$P = P = \frac{G}{i^2}\left[\frac{(1+i)^N - Ni - 1}{(1+i)^N}\right] = G(P/G, i, N)$$

The resulting factor in brackets is called the gradient series present-worth factor, which we denote as (P/G, i, N)

To find F,

$$F = G\left[\frac{(1+i)^N - 1}{i^2}\right] - \frac{NG}{i}$$

Functionally

$$G(F/G, i, N) - \frac{NG}{i}$$

To find A,

$$A = G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]$$

Functionally, G (A/G, i N)

4. **Geometric Gradient series:**

When the series in cash flow is determined not by a fixed amount like above but by some fixed rate, then the series is called Geometric gradient series.

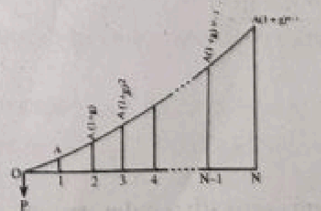


Fig. Geometric Gradient Series

Here, $F_1 = A(1+i)^{N-1}$

$F_2 = A(1+g)(1+i)^{N-2}$

$F_3 = A(1+g)^2(1+i)^{N-3}$

$F_N = A(1+g)^{N-1}$

So, $F = F_1 + F_2 + \ldots\ldots\ldots + F_N$

$= A(1+i)^{N-1} + A(1+g)(1+i)^{N-2} + A(1+g)^2(1+i)^{N-3} + \ldots A(1+g)^{N-1}$

(From sequence and series formula)

$$\text{Sum} = \frac{a(1-r^n)}{1-r} \qquad [\because r = \text{common ratio}]$$

$$F = A(1+i)^{N-1}\frac{\left[1-\left(\frac{1+g}{1+i}\right)^N\right]}{1-\left(\frac{1+g}{1+i}\right)} \text{ here } r = \left(\frac{1+g}{1+i}\right)$$

$$= A(1+i)^{N-1}\frac{[(1+i)^N-(1+g)^N]}{1+i-1-g}\times\frac{(1+i)}{(1+i)^N}$$

$$= A\left[\frac{(1+i)^N-(1+g)^N}{i-g}\right]$$

$$\therefore \boxed{F = A\left[\frac{(1+i)^N-(1+g)^N}{i-g}\right]}$$

If $i = g$ then, $F = NA(1+i)^{N-1}$

To find p,

$$P = \frac{A[1-(1+g)^N(1+i)^{-N}]}{(i-g)}$$

If $i = g$

$$p = \left[\frac{NA}{1+i}\right]$$

5. **Unequal payment series/Irregular series:** A series of cash flow which are irregular and doesn't exhibit an overall regular pattern.

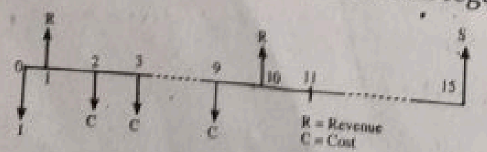


Fig. Uneven Series

Some formulae:

A. **Equal payment series:** $F = A\left[\frac{(1+i)^N-1}{i}\right]$

B. **Linear gradient series:** $F = \frac{G}{i}\left[\frac{(1+i)^N-1}{i}\right] - \frac{NG}{i}$

$$A = G\left[\frac{1}{i} - \frac{N}{(1+i)^N-1}\right]$$

& $$P = \frac{G}{i^2}\left[\frac{(1+i)^N - Ni - 1}{(1+i)^N}\right]$$

C. **Geometric gradient series:** $F = A\left[\frac{(1+i)^N-(1+g)^N}{i-g}\right]$

D. **For compounding:**

Rule 1: From higher compounding period to lower compounding period.

Eg: $i_{quarter} = \frac{i_{year}}{4}$ or $i_{quarter} = \frac{i_{semi}}{2}$

$i_{semi} = \frac{i_{year}}{2}$

Rule2: From lower compounding period to higher compounding period.

eg: $i_{semi} = (1+i_{quarter})^2 - 1$

$i_{year} = (1+i_{month})^{12} - 1$

For withdrawn: if i = interest rate per year and withdraw is monthly then $i_{month} = (1+i_{year})^{1/12} - 1$

6. **Continuous compounding and continuous compounding formulas.**

a) Continuous compounding and discrete cash flow

Here cash flows occurs at discrete interval (eg. once per year) but it is compounded continuously throughout the interval

Let the nominal rate of interest per year be r, and the use of continuous compounding

$i = e^r - 1$

We have, $F = P(1+i)^N$

$F = Pe^{rN}$

$F = P(F/P, r\%, N)$

Interest Factors and Symbols

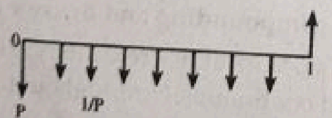
To Find	Given	Factor	Factor Name	Functional Symbol
Single cash flow				
F	P	e^{rN}	Continuous compounding compound factor	{F/P, r%, N}
P	F	e^{-rN}	Continuous compounding present equivalent	{P/F, r%, N}
Uniform series				
F	A	$\frac{e^{rN}-1}{e^{r}-1}$	Continuous compounding compound amount	{F/A, r%, N}
P	A	$\frac{e^{rN}-1}{e^{rN}(e^{r}-1)}$	Continuous compounding present equivalent	{P/A, r%, N}
A	F	$\frac{e^{r}-1}{e^{rN}-1}$	Continuous compounding sinking fund	{A/F, r%, N}
A	P	$\frac{e^{rN}(e^{r}-1)}{e^{rN}-1}$	Continuous compounding capital recovery	{A/P, r%, N}

b) Continuous compounding and continuous cash flow

Continuous cash flow means a series of cash flows occurring at infinitesimally short interval.

Let r = nominal interest per year

If there are P numbers of payment per year which amount to a total of one unit per year, then



We have,

$$F = A\{((1+i)^N - 1)/i\}$$

$$F = A\{((1+i) - 1)/i\} \text{ (if } N = 1)$$

Future equivalent at the end of year 1

$$F = \frac{1}{p}\{((1+r/p)^p - 1)/(r/p)\}$$

$$F = \{((1+r/p)^p - 1)/r\}$$

As $F = P(1+r/p)^p$

$$P = \frac{(1+r/p)^p - 1}{r(1+r/p)^p}$$

As $\lim_{P\to\infty}(1+r/p)^p = e^r$

Present equivalent of continuous one year cash flow

$$P = 1\{e^r - 1/r(e^r)\}$$

$$P = e^r - 1/re^r$$

$$(P/A, r\%, N) = er^N - 1/re^{rN}$$

Where A = amount flowing uniformly and continuously over one year

Thus,

i) $P = A\{(e^{rN} - 1)/re^{rN}\}$ ii) $F = A\left\{\frac{er^N - 1}{r}\right\}$

Solved Example

1. **A person deposits a sum of Rs 5000 in a bank at a nominal interest rate of 12% for 10 yrs. The compounding is quarterly. Find maturity of deposit after 10 yrs.**

Solution:

Given, P = Rs 500, N = 10 yrs, r = 10% compounded quarterly

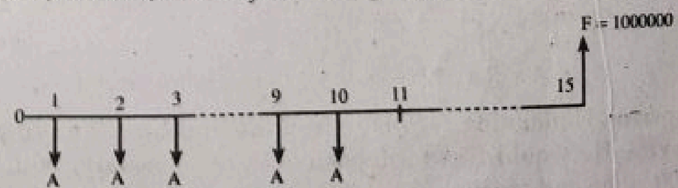
So, $i_{effective} = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{0.12}{4}\right)^4 - 1 = 12.5508\%$ per year.

$$F = P(1+i)^N = 5000\,(1+0.1255)^{10} = \text{Rs } 16310$$

2. **Mr. Jha wants to have \$ 1000000 for the studies of his daughter after period of 15 yrs. How much rupees does he has to deposit each year for 10 continuous yrs in a saving account that earns 8% interest annually.**

Solution:

Given: F = \$1000000, N = 15 yrs i = 8% per year



Find: A First discounting F = 1000000 to year 10

$F_{10} = F \times (1 + i)^{-N} = 1000000 \times (1 + 0.08)^{-5} = \$\,680,583.19$

Now, from equal payment series formula

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] \quad (\because F \text{ is } F_{10})$$

$$\therefore A = F \times \frac{i}{(1+i)^N - 1}$$

$$= 680583.19 \times \frac{0.08}{(1.08)^{10} - 1} = \$\,46,980.3$$

3. **A man aged 40 years now had borrowed Rs 500000 from a bank for his future studies at the age of 20 yrs. Interest was charged at 11% per year compounded quarterly. He wished to pay loan in semiannual equal installments with the first installment beginning 5 yrs after receiving loan. He just cleared the loan now. What amount did he pay in each installment?**

Solution:

Given, P = Rs. 500000 I = 11% per year & compounding quarterly.

N = 20 years A = ?

Quarterly interest rate,

$$i_q = \frac{i_{year}}{4} = \frac{11\%}{4} = 2.75\% \text{ [From formula No.4]}$$

Semi-annual interest rate

$$i_{semi} = (1 + i_q)^2 - 1 = (1 + 0.0275)^2 - 1 = 5.57\%$$

Using single payment compound amount factor

$$F = P(1+i)^N = 500000(1 + 0.057)^{40} \quad \text{.........................(1)}$$

Using uniform series compound amount factor

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A\left[\frac{(1+0.057)^{30} - 1}{0.057}\right] \quad \text{.............(2)}$$

Equating equation (i) & (ii)

$$500000\,(1.057)^{40} = A\left[\frac{(1.057)^{30} - 1}{0.057}\right]$$

$\therefore$ A = Rs. 61217.32 is semi-annual payment.

4. **A person is planning for his retired life and has 10 more years of service. He would like to deposit 30% of his salary, which is Rs 5000 at the end of first year and thereafter he wishes to deposit the**

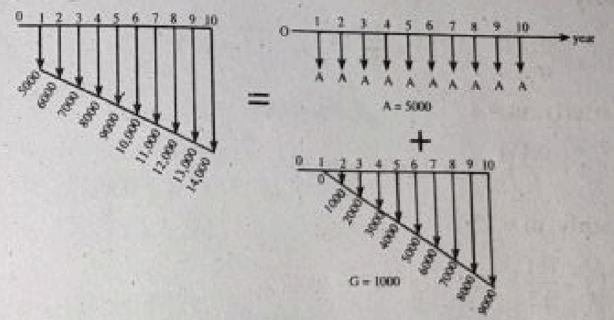
amount with an annual increase of Rs. 1000 for next 9 yrs with an interest rate of 15%. Find the total amount at end of 10th year with the above series.

Solution:

Given: (It is the problem of Linear gradient series)

G = Rs. 1000, A = Rs. 5000, N = 10 yrs and i = 15% per year

Find: F = future worth



$F = F_A + F_G$

F_A = Future worth from Equal payment series

F_G = Future worth from Linear gradient series

$$\text{So, } F = A\left[\frac{(1+i)^N - 1}{i}\right] + \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$$

$$= 5000\left[\frac{(1+0.15)^{10} - 1}{0.15}\right] + \frac{1000}{0.15}\left[\frac{(1+0.15)^{10} - 1}{0.15}\right] - \frac{10 \times 1000}{0.15}$$

= Rs. 170209.33

5. **An investment of Rs. 100,000 is made in a company. The first year of the investment produce net revenue of Rs. 20,000. Over a 40 year period, the net revenue received from the investment decreased by 10% each year. If the interest rate is 12%, what is the present worth for the investment?**

Solution:

Given A = 20,000, I = 12%, g = –10%, N = 40 years.

$$Pw = -100000 + 20000\left[\frac{1 - (1-g)^N (1+i)^N}{i - g}\right]$$

$$= -100000 + 20000\,\frac{(1 - 0.9^{40} \times 1.12^{-40})}{0.12 - (-0.1)} = \text{Rs. } 13,618.3$$

6. **What is the effective interest rate of nominal interest rate 10% per year if the compounding is**
   a) Yearly **b) quarterly**
   c) monthly **d) daily**
   e) continuously

Solution: For compound

a) **Yearly**

$$i = \left(i + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.1}{1}\right)^1 - 1 = 0.1 = 10\%$$

b) **Quarterly, m = 4**

$$i = \left(1 + \frac{0.1}{4}\right)^4 - 1 = 0.1038 = 10.38\%$$

c) **Monthly, m = 12**

$$i = \left(1 + \frac{0.1}{12}\right)^{12} - 1 = 0.1047 = 10.47\%$$

d) **Daily, m = 365**

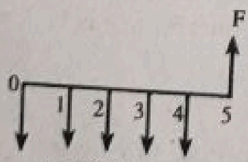
$$i = \left(1 + \frac{0.1}{365}\right)^{365} - 1 = 0.1051 = 10.51\%$$

e) **Continuously, m = ∞**

$$i = \left(1 + \frac{0.1}{\infty}\right)^{\infty} - 1 = e^r - 1 = e^{0.1} - 1 = 0.10517 = 10.517\%$$

7. **Determine the FW of given cash flow data.**
   Deposited at 8% compounded continuously for five years.
   i) Rs. 50,000 at the beginning of each year
   ii) Rs. 50,000 at the end of each year

Solution:



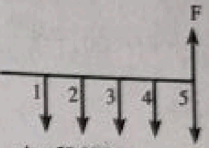
Using continuous compound amount factor (at the end of 4th year)

$$F = A\,(e^{rN} - 1)/(e^r - 1)$$
$$= 50{,}000 \times (e^{0.08 \times 5} - 1)/(e^{0.08} - 1)$$
$$= \text{Rs. } 295258.74$$

So, P at 4th year = 295258.74

$$F = Pe^{rN}$$
$$= 295258.74 \times e^{0.08*1}$$
$$= 319849.98$$

Again,



$$F = A\,(e^{rN} - 1)/(e^r - 1)$$
$$= 50{,}000 \times (e^{0.08 \times 5} - 1)/(e^{0.08} - 1)$$
$$= \text{Rs. } 295258.74$$

Old Question Solution

1. **Ramesh, a civil engineer is planning to plane a total of 20% of his salary, which is Rs 250,000 per year. He expects 7% increase in salary for next 15 years if the mutual fund results in 10% annual return, what will be the amount at the end of 15 years. If salary increases by 25000/ year, what will be the amount? [2069 Bhadra]**

Solution:

For the first case, the flow is geometric

A = 20% of 250,000 = 50000

$g = 0.07$

$i = 0.10$

$N = 15$

We have,

$$P = A \times \frac{1 - (1+g)^N (1+i)^{-N}}{i - g}$$

$$= 50000 \times \frac{1 - (1.07)^{15} \times 1.10^{-15}}{0.10 - 0.07}$$

$$= 565849.642$$

$$F = P(1+i)^N = 2363694.381$$

b) If salary increases by Rs.25,000/year than

$A = 50000$

$G = 5000/\text{year}$

$$F = 50000\,(F/A,\ 10\%,\ 15) + \frac{5000}{0.10}\left(\frac{1.10^{15}-1}{0.10}\right)\frac{-15\times 5000}{0.10}$$

$$= 50000\times\frac{1.10^{15}-1}{0.10} + 50000\left(\frac{1.10^{15}-1}{0.10}\right)\frac{-15\times 5000}{0.10}$$

$F = \text{Rs. } 24{,}27{,}248.164$

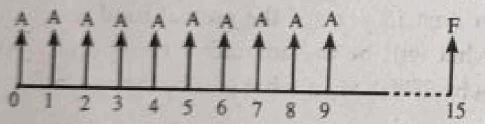
2. **Define nominal and effective interest rate.** **[2071 Bhadra]**

Solution:

See at 2.3.1 and 2.3.2

3. **Evaluate FW at the end of 15 yrs with 10% interest rate compounded monthly on a cash flow of 50,000 at the beginning of each first 10 years.** **[2071 Bhadra]**

Solution:



Given, $A = 50000,\ r = 10\%$

$$i = \left(1+\frac{r}{m}\right)^{m} - 1$$

$$= \left(1+\frac{0.1}{12}\right)^{12} - 1$$

$= 0.1047$

$= 10.47\%$

At the end of 9^{th} year or begging of 10^{th} year

$$FW = A\left[\frac{(1+i)^N-1}{i}\right]$$

$$= 50000\left[\frac{(1+0.1047)^{10}-1}{0.1}\right]$$

$= \text{Rs. } 853360.65$

So, the value of FW at the end of 15^{th} year is

$F = P(1+i)^N$

$= 853360.65\,(1+0.1047)^6$

$= \text{Rs. } 1550953.35$

4. **What will be the amount of money at the end, if you deposit Rs. 5000 per month for five years continuously? If nominal interest rate is 10% & compounded quarterly.** **[2071 Magh]**

Solution:

Given, A = Rs. 5000 per month for five yrs

$i_{nominal}$ = 10% & compounded quarterly

so, $i_{quarter} = \frac{i_{year}}{4} = \frac{10}{4} = 2.50\%$

$i_{monthly} = (1 + i_{quarter})^{1/4} - 1$

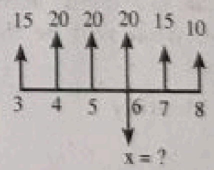
$= 0.62\%$

$$\text{So, } FW = A\left[\frac{(1+i)^N-1}{i}\right]$$

$$= 5000\left[\frac{(1+0.0062)^{60}-1}{0.0062}\right]$$

$= 362070.93$

5. **Calculate the value of x from the following figure. (i = 10%)** **[2071 Magh]**



Solution:

Compounding 15, 20, 20, 20 to year-6 and discounting 15 & 10 to year-6

So that,

$$15(1+0.1)^3 + 20\left[\frac{(1+i)^3-1}{i}\right] + \frac{15}{(1+0.1)} + \frac{10}{(1+0.1)^2} - x = 0$$

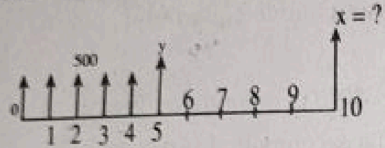
$19.965 + 66.2 + 13.636 + 8.264 - x = 0$

$\therefore x = 108.065$

6. **What is the effective and nominal interest rate? Evaluate FW at the end of 10 yrs with 8% interest rate compounded continuously with cash flow of Rs. 500 at the beginning of each year for first five years.** **[2070 Bhadra]**

Solution:

For theory part See at 2072 Ashwin



Transforming the amount to end of each year

$T = 500 \times e^{0.08}$

The worth of all at the end of 5th year is

$Y = 500 \times e^{0.08}$ (F/A, 8%, 5)

$= 500 \times e^{0.08} \times \frac{e^{0.08\times 5} - 1}{e^{0.08} - 1} = 3198.499$

The worth of it at the end of 10 years is

$x = 3198.439$ (F/e, 8%, 5)

$= 3198.439 \times e^{0.08\times 5}$

$= 4771.6$

7. **If you deposit Rs 10,000 is a saving account now, which gives 10% nominal interest rate, what will be the amount after 5 years if interest is compounded**

(i) Semi annually (ii) Monthly **[2070 Magh]**

Solution:

i) If interest is compounded semi-annually then m = 2 and the effective interest rate is

$i_{eff} = (1 + r/m)^m - 1$

$= \left(1 + \frac{0.10}{2}\right)^2 - 1$

$= 10.25\%$

$F = P$ (F/P, 10.25%, 5)

$= 10000(1.1025)^2$

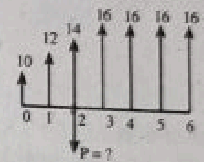
= Rs. 16288.952

ii) For monthly compounding

$i_{eff} = \left(1 + \frac{0.10}{12}\right)^{12} - 1 = 10.47\%$

$F = P(F/P, 10.47, 5) = 10000 \times 1.1047^5 = \text{Rs. } 16452.116$

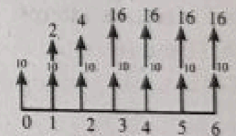
8. **Find the value of P if i = 10%, use gradient formula also.** **[2070 Magh]**



Solution:

Discounting all cash flow to year 2,

$P = 10(F/A, 10\%, 3) + [\frac{G}{i}(F/G, 10\%, 3) - \frac{NG}{i}] + 16(P/A, 10\%, 4)$



$P = 10 \times \frac{(1+i)^N - 1}{i} + \left[\frac{2}{0.10} \times \frac{(1+i)^N - 1}{i} - \frac{NG}{i}\right] + 16 \times \frac{(1+i)^N - 1}{i(1+i)^N}$

$= 10 \times \frac{1.10^3 - 1}{0.10} + \left(\frac{2}{0.10} \times \frac{1.10^3 - 1}{0.10} - \frac{3\times 2}{0.10}\right) + 16 \times \frac{1.10^4 - 1}{0.10 \times 1.10^4}$

= Rs. 90.004

9. **What is difference between nominal and effective interest rate?** **[2072 Ahswin]**

Solution:

Nominal: Interest rates are the stated, advertised or quoted rates where no time period is started, than per year (also known as per annum) is assumed.

Effective: Interest rates are what borrow have to actually pay, and depend on how frequently the nominal rate is compounded. (i.e. which means adding interest to the balance of the loan).

For formula and examples: Refer at 2.3.1 and 2.3.2

10. You deposit Rs. 1000 in your bank account. If the bank pays 4% simple interest, how much will you accumulate in your account after 10 years? What if the bank pays compound interest? How much of your earnings will be interest on interest? [2072 Ahswin]

Solution:

Principle (P) = 1000

Simple interest rate (i) = 4%

Time (N) = 10 years

The amount accumulated in bank @ 4% Simple interest = P + PiN

$= 1000 + 1000 \times 0.04 \times 10 = \text{Rs. } 1400$

Amount accumulated in bank @ 4% compound interest,

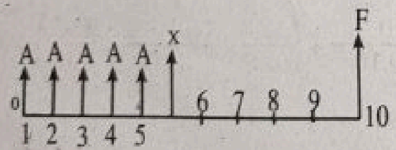
$= P(1 + i)^N = 1000\,(1 + 0.04)^{10} = \text{Rs. } 1480.24$

Interest on interest = 1480.24 – 14000 = Rs. 80.24

11. What is nominal and effective interest rate? Evaluate FW at the end of 10 years with 12% interest rate compounded monthly of a cash flow of Rs. 40,000 at the beginning of each year for 5 years. [2073 Bhadra]

Solution:

For Nominal & effective interest rate See at 2.3.1 and 2.3.2



A = 40,000   R = 12%

$$i = \left(1 + \frac{r}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.12}{12}\right)^{12} - 1$$

$$= 0.1268$$

$$= 12.68\%$$

At the end of 5th year or beginning of 6th year

$$X = A\left[\frac{(1 + i)^N - 1}{i}\right]$$

$$= 40000\left[\frac{(1 + 0.127)^6 - 1}{0.127}\right]$$

$$= 330{,}395.98$$

So, the value of FW at the end of 10th year is

$$F = P(1 + i)^N$$

$$= 330{,}395.98\,(1 + 0.127)^5$$

$$= 600{,}695.45$$

12. A person invest a sum of 50,000 in bank at a nominal interest rate of 18% for 15 years. The compounding is monthly. Find the maturity amount of the deposit after 15 years. Also briefly explain the importance of time value of money. [2073 Magh]

Solution:

First Part:

Invest sum = 50,000

Nominal interest rate = 18%

Effect interest rate (i_{eff}) $= \left(1 + \frac{r}{m}\right)^m - 1$

$$= \left(1 + \frac{0.18}{12}\right)^{12} - 1$$

$$= 19.56\%$$

Maturity amount of the deposit after 15 years

$$F = P(F/P, 19.56\%, 15)$$

$$= 50{,}000 * 1.1956^{15}$$

$$= \text{Rs. } 729052.15$$

2nd Part

Time Value of Money (TVM) is an important concept to investors because a dollar on hand today is worth more than a dollar promised in the future. The dollar on hand today can be used to invest and earn interest or capital gain.

A dollar promised in the future is actually worth less than a dollar today because of inflation. The time value of money can be broken up into two areas.

- Present value
- Future value

$$\text{Present value} = \frac{\text{Future value}}{(1 + \text{rate of return})^{\text{number of periods}}}$$

Present value determine what a cash flow to be received in the future is worth in today's dollars. It discounts the future cash flow back to the present date, using the average rate of return and the number of periods. No matter what the present value is, if you invest that present value amount at the specified rate of return and number of periods, the investment would grow into the future cash flow amount.

Similarly, future value = Present value * $(1 + \text{rate of return})^{\text{number of periods}}$

Future value determines what a cash flow received today is worth in the future, based on interest rates or capital gains. It calculates what a current cash flow would be worth in the future, it was invested at a specified rate of return rate of return and number of periods.

Both present value and future value take into account compounding or capital gains, another important aspect for investors looking for good investments.

13. **If you make equal monthly deposits of Rs. 5000 in the bank for 10 years, saving accounts that pays interest rate of 6% compounded monthly, what could be the amount at the end of 15 years?** **[2074 Bhadra]**

Solution:

Here deposit is monthly (A) = 5000

Interest rate = 6% compounded monthly

i.e. $\frac{6}{12} = 0.5\%$ per month

up to 10 years, number of payments (N) = 10 × 12 = 120

A = Rs. 5000

i = 0.5 % = 0.005

$$F_{10} = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 5000\left[\frac{(1+0.005)^{120} - 1}{0.005}\right]$$

$$= 819396.73$$

At the end of 10^{th} year P10 = 819396.73

Yearly effective interest rate (i_a) $= (1 + i_m)^{12} - 1$

$= (1 + 0.005)^{12} - 1$

$= 0.062$

Now, $F_{15} = P_{10}(1 + i_a)^n$ [Here n = 5 years]

$= 819396.73\,(1 + 0.062)^5$

= Rs. 1106921.46

i.e. Amount at the end of 15 years is Rs. 1106921.46

14. **How many rupees should you deposit now so that you will be able to draw Rs. 5000 at the end of this month which increases by 2 percent per month for 15 years? Bank interest rate is 5% per year.** **[2074 Bhadra]**

Solution:

Given, Yearly interest rate (i_a) = 5%

Interest on first month i.e. (A) = 5000

Geometric Gradient (g) = 2% = 0.02

N = 15 years = 15 × 12 = 180 months

Monthly interest rate $(i_m) = \frac{5}{12}\% = 0.417\% = 0.00417$

Now, Using Geometric gradient formula

$$F = A\left[\frac{(1+i)^N - (1+g)^N}{i - g}\right]$$

$$= 5000\left[\frac{(1+0.00417)^{180} - (1+0.02)^{180}}{0.00417 - 0.02}\right]$$

$$= 5000 \times \frac{33.205}{0.016}$$

$$= 10376562.50$$

Now,

$$P = F(1+i)^{-N}$$

$$= 10376562.50\,(1 + 0.00417)^{-180}$$

$$= 4906252.05$$

So you have to deposit Rs. 4906252.05

15. **Define time value of money, nominal interest rate and effective**

interest rate. Calculate the future sum at the end of 5th year when monthly deposit is Rs. 6000 for 3 years that earns 7% interest per year. **[2075 Baisakh]**

Solution:

For definition refer to chapter content.

Given, monthly deposit (A) = Rs. 6000 up to 3 years

i.e. $n = 3 \times 12 = 36$

Yearly interest = 7%

Monthly interest $(i_m) = \frac{7}{12}\% = 0.583\% = 0.00583$

At the end of 3 years $F_3 = A\left[\frac{(1+i)^n - 1}{i}\right]$

$= 6000\left[\frac{(1+0.00583)^{36} - 1}{0.00583}\right]$

$= 6000 \times 39.927$

$= 239566.21$

Now, At 3rd year $P_3 = 239566.21$

$i = 7\% = 0.07$

For 5th year $n = 2$ years

$F_5 = P_3 (1+i)^n$

$= 239566.21 (1 + 0.07)^2$

$= 274279.35$

That is future sum at the end of 5th year is Rs. 274279.35

16. **A machine needs uniform semi-annual cash flow of $ 10,000 for fuel for 5 years. If interest rate is 12% compounded quarterly, what is its equivalent present worth?** **[2076 Bhadra]**

Solution: *Refer 2071 Magh*

3 BASIC METHODOLOGIES OF ENGINEERING ECONOMICS ANALYSIS

In most of the practical decision environments, executives will be forced to select the best alternative from a set of competing alternatives. Let us assume that an organization has a huge sum of money for potential investment and there are no. of different projects whose initial outlay and annual revenues during their lives are known. The executive has to select the best alternative among these competing projects.

There are several bases for comparing the worthiness of the projects. These bases are:

1. Present worth method
2. Future worth method
3. Annual equivalent method
4. Rate of return method

3.1 Determining Minimum Attractive (Acceptable) Rate of Return (MARR)

MARR is the interest rate at which firm can always earn or borrow money. MARR can also be regarded as the minimum return required to get investors to put up the money. MAAR can be developed from existing projects and may be different from time to time within the same firm.

Factors influence the determination of MARR

1. The amount of fund available for investment and its source.
2. The nature of investment alternatives.
3. The amount of risk perceived in the investment.
4. The type of organization involved. (Government, public, private)

- MAAR is determined from the opportunity cost viewpoint.

Example: Consider the following schedule, which shows prospective annual rates of profit for a company's portfolio of capital investment projects.

Expected Annual Rate of Profit	Investment Requirements	Cumulative Investment
4% and over	Rs. 2,200	Rs. 2,200
30- 39.9%	3,400	5,600
20- 29.9%	6,800	12,400
10 - 19.9%	14,200	26,600
Below 10%	22,800	49,400

If the supply of capital obtained from internal and external sources has a cost of 15% per year for the first Rs. 5,000,000 invested and then increases 1% for every Rs. 5,000,000 thereafter, what is this company's MAAR when using an opportunity cost viewpoint?

Solution: Cumulative capital demand versus supply can be plotted against prospective annual rate of profit, as shown in figure. The point of intersection is approximately 18% per year, which represents a realistic estimate of this company's MARR when using the opportunity cost view point.

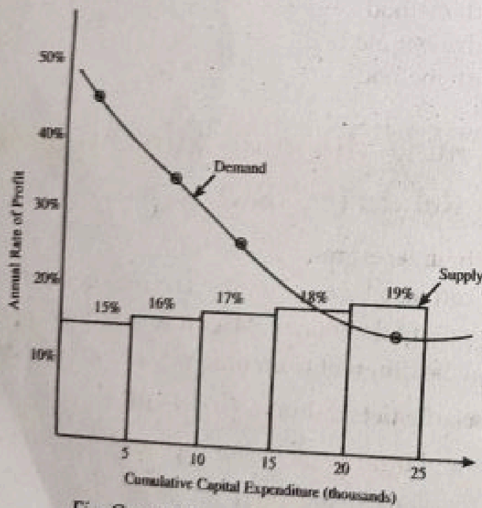


Fig: Cumulative capital expenditure

3.2 Payback Period method

Payback period is the length of time needed before an investment makes enough to recoup the initial investments. The payback method calculates the number of years required for cash inflows to just equal cash outflows.

Based on the way we compute the payback period, it can be classified in to two types.

1. Simple or conventional payback period.
2. Discounted payback period.

1. Simple payback period (SP)

Simple payback period doesn't consider the time value of money (i = 0) It measures a projects liquidity and is computed as follows.

$$SP = \frac{\text{Initial Investemt}}{\text{Expected savings or Reveneue Per year}}$$

OR,

$$SP = \text{years before full recovery} + \frac{\text{unrecovered cost at start of year}}{\text{cash flow drawing following year}}$$

In other way, SP period for a project having one time investment at time 0,

$$\sum_{k=1}^{\theta} (R_k - E_k) - I > o$$

Where, I = Capital investment.

R_k = Revenues at the end of year k.

E_k = Expenses at the end of year k.

θ = Simple payback period.

Benefits and Flaws of Payback Period

The simplicity of the payback method is one of its most appealing qualities. Initial project screening by the method reduces the information search by focusing on that time at which the firm expects to recover the initial investment. The method may also eliminate some alternatives, thus reducing the firm's time spent analyzing. But the much-used payback method of equipment screening has a number of serious drawbacks.

The principal objection to the method is that it fails to measure profitability (i.e., no "profit" is made during the payback period). Simply measuring how long it will take to recover the initial investment outlay contributes little to gauging the earning power of a project. (In other words, you already know that the money you borrowed for the drill press is costing you 12% per year; the payback method can't tell you how much your invested money is contributing toward the interest expense.) Also, because payback period analysis ignores differences in the timing of cash flows, it fails to recognize the difference between the present and future value of money. For example, although the payback on two investments can be the same in terms of numbers of years, a front-loaded investment is better because money available today is worth more than that to be gained later. Finally, because payback screening ignores all proceeds after the payback period, it does not allow for the possible advantages of a project with a longer economic life.

In summary advantage and dis-advantage can be presented as

Advantages

- Simplicity
- It doesn't require any assumption.
- Reduces information search by focusing on that time at which the firm expects to recover the initial investment.

Disadvantages

- Fails to measure profitability
- It takes no account of the time value of money.
- It takes no account of the residual value in the capital asset.

Example: Calculate simple payback period for the given cash flow of the project.

End of Year (EOY)	Net cash flow (Rs)
0	- 25,000
1	8,000
2	8,000
3	8,000
4	8,000
5	13,000

Solution:

End of Year (EOY)	Net cash flow (Rs)	Cumulative cash flow (Rs)
0	- 25,000	- 25, 000
1	8,000	- 17, 000
2	8,000	- 9, 000
3	8,000	- 1, ,000
4	8,000	+ 7, 000
5	13,000	+ 20, 000

Here, the cumulative cash flow turns to positive in year 4. Therefore payback period lie between year 3 and 4. By interpolation,

Payback period = 3.125 years

[**Note:** *Interpolation Technique by fx – 991ES plus calculation*

Mode ⇒ 3 (STAT) ⇒ 2 (A + Bx) ⇒

For example: x   *y*

- 1, 000   *3*

+7, 000   *4*

⇒ AC ⇒ SHIFT ⇒ 1 (STAT) ⇒ 5 (Reg.) ⇒

$5(\hat{y})$ ⇒ (Interpolation value) $\hat{y}$

$0\hat{y} = 3.125$]

2. **Discounted payback period :** It is defined as the number of years required to recover the investment from discounted cash flows. I.e. considering the time value of money.

Discounted payback period for a project having one time investment at time zero can be computed as

$$\sum_{k=1}^{\theta} (R_k - E_k)(P/F, i\%, k) - I > 0$$

Where, I = capital investment

R_k = Revenues at the end of year k.

E_k = Expense at the end of year k.

i = Minimum Attractive Rate of return.

θ = Discounted payback period.

Advantage

- Consider the time value of money.
- Consider the riskiness of the project's cash flow.

Disadvantage

- Ignores cash flows beyond the discounted payback period.
- Requires an estimate of the cost of capital in order to calculate the payback.
- No concrete decision criteria that indicate whether to investment increases the firm's value.

Example

Evaluate the acceptability of the following proposal if maximum

allowable discounted payback is 6 years. Assume MARR = 15%

EOY	0	1	2	3	4	5	6	7
Amount (in thousand)	-1500	200	400	450	450	600	900	1100

Solution:

EOY	Net cash how	PW at 15%	Cumulative PW
0	- 1500	-1500	-1500
1	200	$200/(1+0.15)^1 = 173.91$	-1326.09
2	400	$400/(1+0.15)^2 = 302.46$	-1023.63
3	450	$450/(1+0.15)^3 = 295.88$	-727.747
4	450	257.29	-470.458
5.	600	298.31	-172.152
6	900	389.09	216.9431
7	1100	413.53	630.47

From the above table, cumulative PW at 15% has - ve sign at the end of 5 year and + ve at the end of 6 year. Therefore, the discounted payback period lies in between 5 and 6 years.

By interpolation

-172.152	5
216. 943	6
0	? (5.44) using calculator.

Decision:

Discounted payback period = 5.44 < 6 years

The proposal is acceptable.

3.3 Equivalent Worth Methods

3.3.1 Present worth method.

Present worth method discounts future amounts to the present by using the interest rate over the appropriate study period.

We will first summarize the basic procedure for applying the net-present-worth criterion to a typical investment project:

- Determine the interest rate that the firm wishes to earn on its investments. The interest rate you determine represents the rate at which the firm can always invest the money in its **investment pool**.

This interest rate is often referred to as either a **required rate of return** or a **minimum attractive rate of return** (MARR). Usually, selection of the MARR is a policy decision made by top management. It is possible for the MARR to change over the life of a project, but for now we will use a single rate of interest in calculating the NPW.

- Estimate the service life of the project.
- Estimate the cash inflow for each period over the service life.
- Estimate the cash outflow over each service period.
- Determine the net cash flows (= Inflow - Outflow)
- Find the present worth of each net cash flow at the MARR. Add up all the present worth figures; their sum is defined as the project's NPW, given by

$$PW(i\,\%) = F_0(1+i)^0 + F_1(1+i)^{-1} + F_2(1+i)^{-2} + \ldots\ldots + F_N(1+i)^{-N}$$

$$= \sum_{k=0}^{N} F_k(1+i)^{-k}$$

Where, i = effective interest rate or MAAR

k = index $(0 \le k \le N)$

F_k = future cash flow of at the end of period k

N = Number of compounding periods in study period.

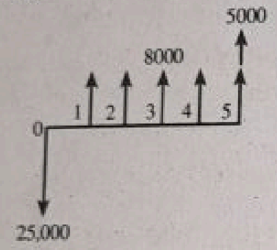
While evaluating a project of by PW method, the following rule appreciable

- **Single Project Evaluation.** In this context, a positive NPW means that the equivalent worth of the inflows is greater than the equivalent worth of outflows, so the project makes a profit. Therefore, if the PW (*i*) is positive for a single project, the project should be accepted; if the PW (*i*) is negative, the project should be rejected. The decision rule is
  If PW (i %) > 0, accept the project.
  If PW (i %) = 0 remain indifferent.
  If PW (i %) < 0, reject the project.
- **Comparing Multiple Alternatives.** Compute the PW (*i*) for each alternative and select the one with the largest PW (*i*). When we compare mutually exclusive alternatives with the *same revenues*, they are compared on a *cost-only basis*. In this situation (because you are minimizing costs, rather than maximizing profits), we should accept

the project that results in the *smallest*, or *least negative*, NPW.

Example: A piece of new equipment has been proposed by engineers to increase the productivity of certain manual welding operation. The investment cash is Rs. 25,000 and the equipment will have a market value of 5,000 at the end of a study period of Five years, increased productivity attributable to the equipment will amount to Rs.8, 000 per year after extra operating costs have been subtracted from the revenue generated by the additional production, is this proposal a sound one? MAAR = 20%, use PW method.

Solution: Cash flow diagram



Net PW = PW of cash in inflows – PW of cash flows

$= 8{,}000\,(P/A, 20\%, 5) + 5{,}000\,(P/F, 20\%\ 5) - 2500$

$= 8{,}000\dfrac{[(1+0.2)^5 - 1]}{0.2 \times (1+0.2)^5} + 5{,}000 \times \dfrac{1}{(1+0.2)^5} - 2500$

$= 23924.89 + 2009.38 - 25{,}000 = \text{Rs } 934.27$

PW (20 %) > 0 this equipment is economically justified.

3.3.2 Future worth method

Future worth of a project is the equivalent worth of all cash flows, both inflows and outflows, at the end of the planning horizon of an interest rate that is generally the MAAR. The FW is calculated as,

$$FW(i\%) = F_0\,(1+i)^N + F_1\,(1+i)^{N-1} + \ldots\ldots + F_N\,(1+i)^0$$

$$= \sum_{k=0}^{N} F_k\,(1+i)^{N-k}$$

Where, I = effective interest rate or MAAR

k = index for each compounding period.

F_k = future cash flows at the end of period k.

N = Number of compounding periods in study period.

Decision rule:

If FW (i %) > 0, accept the project.

If FW (i %) = 0 remain indifferent.

If FW (i %) < 0, reject the project.

Example: Evaluate the FW of the potential improvement project describe in above example. Show relationship between FW and PW for this example.

Solution:

Net FW (20%) $= -25000\,(F/P, 20\%,5) + 8{,}000(F/A, 20\%5) + 5{,}000$

$= -25{,}000 \times (1+0.2)^5 + 8{,}000\dfrac{[(1+0.2)^5 - 1]}{0.2} + 5000$

= R.S. 2324.8

As, NFW (20%) > 0, this equipment is economically justified

PW (20%) $= 2{,}324.8\,(P/F, 20\%, 5)$

$= \dfrac{2324.8}{(1+0.2)^5} = 934.29$

Same result obtained.

3.3.3 Annual worth method

The **annual equivalent worth (AW)** criterion provides a basis for measuring the worth of an investment by determining equal payments on an annual basis. Knowing that any lump-sum cash amount can be converted into a series of equal annual payments, we may first find the net present worth (NPW) of the original series and then multiply this amount by the capital recovery factor:

$$AW(i) = PW(i)\,(A/P, i, N)$$

Annual worth is the equivalent worth of a lump-sum amount converted into a series of equal payments at the end of each period and is calculated as,

AW (i %) = R – E – CR (i %)

Where, R = equivalent revenues

E = equivalent expenses

CR (i %) = capital recovery amount

Decision rule:

- **Single-project evaluation:** The accept-reject selection rule for a single *revenue* project is as follows:

If AW (i %) > 0, accept the project

If AW (i %) = 0, remain indifferent

If AW (i %) < 0, reject the project

- **Comparing mutually exclusive alternatives:** As with present-worth analysis, when you compare mutually exclusive *service* projects whose revenues are the same, you may compare them on a *cost*-only basis. In this situation, the alternative with the minimum *annual equivalent cost* (or least negative annual equivalent worth) is selected

Applicable of AW

- Asset replacement
- Breakeven analysis
- Make or buy decision
- studies dealing with manufacturing costs
- economic value added analysis (EVA)

Capital Recovery amount

The annual equivalent of the capital cost covering the loss in the value of the asset and interest on invested capital is known as capital recovery amount. Capital recovery amount is calculated as,

CR (i %) = I (A/P,i%,N) – S(A/F, i%,N)

Where, I = initial investment for the period

S = salvage value at the end of the study period

N = project study period

Benefits of AW: In the real world, a number of situations can occur in which AE analysis is preferred, or even demanded, over NPW analysis. For example, corporations issue annual reports and develop yearly budgets. For these purposes, a company may find it more useful to present the annual cost or benefit of an ongoing project, rather than its overall cost or benefit. Following are some additional situations in which AE analysis is preferred:

1. **Consistency of report formats.** Financial managers commonly work with annual rather than overall costs in any number of internal and external reports. Engineering managers may be required to submit project analyses on an annual basis for consistency and ease of use by other members of the corporation and stockholders.
2. **Need for unit costs or profits.** In many situations, projects must be broken into unit costs (or profits) for ease of comparison with alternatives.
3. **Life-cycle cost analysis.** When there is no need for estimating the revenue stream for a proposed project, we can consider only the cost streams of the project. In that case, it is common to convert this life-cycle cost (LCC) into an equivalent annual cost for purposes of comparison. Industry has used the LCC to help determine which project will cost less over the life of a product. LCC analysis has had a long tradition in the Department of Defense, having been applied to virtually every new weapon system proposed or under development

Example: Solve the same above problem by AW method.

Solution:

$$\text{Net AW }(20\%) = \underbrace{8000}_{R-E} - \underbrace{[25000\,(A/P, 20\%, 5) - 5000\,(A/F, 20\%, 5)]}_{CR}$$

$$= 8000 - 25000 \times \frac{0.2 \times (1+0.2)^5}{[(1+0.2)^5 - 1]} + 5000 \times \frac{0.2}{[(1+0.2)^5 - 1]} = \text{Rs. } 312.4$$

NAW (20%) > 0, economically justified.

3.4 Rate of Return Method

3.4.1 Internal Rate of return method

Internal rate of return of a project is the breakeven interest rate, i, at which equivalent worth of the project's cash flow is zero. This method solves for the interest rate that equates the equivalent worth of an alternative's cash flows (receipts or saving) to the equivalent worth of cash outflows (expenditures, including investment costs).

Based on PW formulation to find out this interest rate, the corresponding equation becomes,

$$PW\,(i\,\%) = \sum_{K=0}^{N} R_k\,(P/F, i\%, k) - \sum_{K=0}^{N} E_k\,(P/F, i\%, k) = 0$$

Where,

R_k = revenues for the K^{th} year

E_k = expenditures (including investment) for the K^{th} year

N = Project life or study period

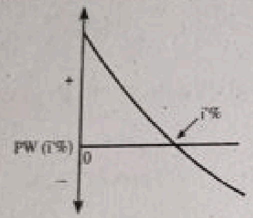
Similarly at the IRR,

FW (i %) = 0

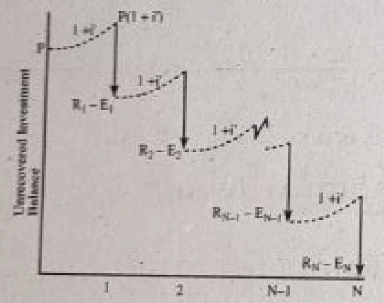
AW (i %) = 0

Though IRR can be computed by equating any equivalent worth to zero, it is advisable to use the PW formulation as it is found excessively used.

Graphically representation



Representation by investment balance diagram



Investment balance diagram showing IRR

Decision Rule

If IRR > MARR, Accept the project

If IRR = MARR, Remain indifferent

If IRR < MARR, Reject the project

Method of finding IRR

1) **Direct solution method :** A project with only a two flow transaction or service life of 2 years of return, we can apply direct mathematical solution for determining the rate of return.

Example:

Period	0	1	2	3	4
Project 1	-1000	0	0	0	1500
Project 2	-2000	1300	1500	-	-

Solution:

Project A

Rs. 1000 = Rs. 1500 (P/F, i%, 4)

$1000 = 1500\,(1+i)^{-4}$

$0.6667 = (1+i)^{-4}$

$\ln(1+i) = \ln\frac{0.6667}{-4} = 0.1013$

$1 + i = e^{0.1013}$

$i = 1.1067 - 1$

$i = 10.67\%$

Project B

$PW(i) = -2000 + \frac{1.300}{(1+i)} + \frac{1500}{(1+i)^2} = 0$

$\text{let } \frac{1}{1+i} = x,$

$-2000 + 1300\,x + 1500\,x^2 = 0$

Solving, $x = 0.8 \text{ or } -1.667$

$\therefore\ I = 25\% \text{ for } x = 0.8$

$i = -160\% \text{ for } x = -1.667,$

$\therefore$ The project's i = 25%

2) Trial and error method

Example:

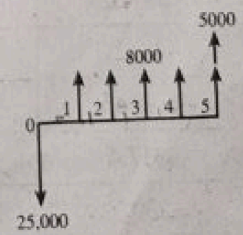
Initial investment = Rs. 25000

Salvage value = Rs. 5000 after 5 years

Net revenue = Rs. 8000 per year MARR = 20%

Is the investment good? Use IRR trial and error method

Solution:



PW(i%) = 0

5000 (P/F, i%, 5) + 8000 (P/A, i%, 5) – 25000 = 0

$$\text{or, } \frac{5000}{(1+i)^5} + 8000 \times \frac{[(1+i)^5 - 1]}{[i \times (1+i)^5]} - 25000 = 0 \quad \text{.........(1)}$$

By trial and error,

At i = 20% PW= 934.3

At i = 25% PW = –1847.1

By interpolation, i = 21.67% at PW = 0

Putting i = 21.67% in equation (1), PW = –58.2

IRR lies between 20% and 21.67%

Again using linear interpolation

i = 21.8%

Putting, i = 21.58% in equation (1)

PW (21.58%) = -1.4

IRR lies between 20% and 21.58%

Again using linear interpolation i = 21.577%

PW (21.577%) ≈ 0

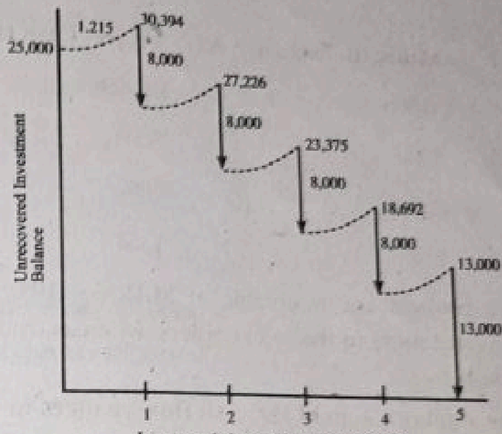
Here,

IRR > MARR (20%), Investment is acceptable

Unrecovered project balance calculation

Year	Unpaid balance at beginning year (Rs.)	Return of unpaid balance (Rs.)	Payment received	Unpaid balance at end of year (Rs.)
0	-25000	0	0	-25000
1	-25000	-25000 × 0.21577 = -5394.25	8000	(-25000 – 5394 + 8000) = -22394.25
2	-22394.25	-4832	8000	-19226.25
3	-19226.25	-4148.45	8000	-15374.7
4	-15374.7	-3317.4	8000	-10692
5	-10692	-2307	13000	0

Unrecovered Balance



Investment balance diagram showing IRR

3. **Computer solution method**

Consider the same example

PW (i %) = 0

8000 (P/A, i%, N) + 5000 (P/F, i%, N) –25000 = 0

$$8000 \times \frac{[(1+i)^5 - 1]}{[(1+i)^5 \times i]} + \frac{5000}{(1+i)^5} - 25000 = 0$$

Write the equation in calculator as,

$8((1 + i)^5–1) / (1 + i)^5 \times i)) + 5/(1 + i) – 25 = 0$

Press Shift ⇒ CALC

⇒The value in the calculator is the value of IRR

Drawbacks of IRR

1. IRR method is based on the assumption that recovered funds, if not consumed in each time period, are reinvested at rather than at MARR. This is not always practical.
2. IRR method involves linear interpolation of non-linear function and when solved manually by trial and error method, may not give accurate results and is more time consuming.
3. There are situations in which its iterative calculation process fails to produce a solution.
4. When naturally exclusive projects are considered it can recommend the wrong investment and does not consider the scale of the investment.

For Example:

EOY	Mutually Exclusive A1	Project Cash flow A2
0	-1000	-5000
1	2000	7000
IRR	100%	40%
PW(10%)	818	1364

Both projects are acceptable at MARR = 10%, but A2 with higher PW is worth more to the stockholders, whereas from IRR point of view, A1 seems better.

5. **When the algebraic sign of the cash flow changes in the middle of the series it is possible to obtain two right answers.**

EOY	Net cash flow
0	-1000
1	2300
2	-1320

We find IRR = 10% and 20% but both of them are in correct. So, use NPW criteria to make the decision.

3.4.2 External/modified rate of return method

IRR has a serious drawback of reinvestment assumption, which is not always practical for cash borrowed or released from a project to be reinvested to yield a rate of return equal to that received from the project. It assumes that net positive cash flows are reinvested at the reinvestment rate ∈ % which depends upon market rate available.

Steps of ERR calculation

1. Discount all expenses to the present at the external reinvestment rate, ∈ %

$$\sum_{K=0}^{N} E_k (P/F, \in\%, k)$$

2. Project all income to the future at the external reinvestment rate ∈ %

$$\sum_{K=0}^{N} R_K (F/P, \in\%, N-k)$$

3. ERR, i% is the interest rate that establishes equivalence between these terms

$$\sum_{K=0}^{N} E_k (P/F, \in\%, k)\,(E/P, i\%\, N) = \sum_{K=0}^{N} R_k (F/P, \in\%, N-k)$$

Where,

R_k = net revenues in period k

E_k = net expenses in period k

N = project's life

∈ % = External reinvestment rate

Decision rule

If ERR > MARR, Accept the project

If ERR = MARR, remain indifferent

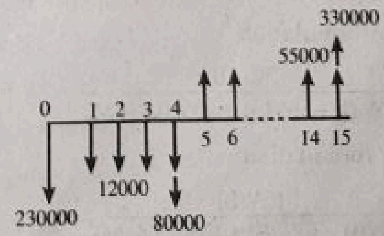
If ERR < MARR, reject the project

Advantages of ERR over IRR

1. It does not need trial and error process to solve for i%
2. There is no possibility of multiple rates of return.

Example: A consultancy opened now for Rs. 230000 will lose Rs.12000 each year the first four years. An additional Rs. 80000 invested in the consultancy during the fourth year will result in a profit of Rs. 55000 each year from the fifth year to the fifteenth year. At the end of 15 years the company can be sold for Rs. 330000. Calculate the external rate of return if external reinvestment rate is12% per year. Is the investment good? MARR = 15%

Solution:



Discount all cash out flow to time zero.

$$230000 + 12000\,(P/A, 12\%, 4) + 80000\,(P/F, 12\%, 4)$$

$$= 230000 + 12000 \times \frac{[(1+0.12)^4 - 1]}{0.12 \times (1+0.12)^4} + \frac{80000}{(1+0.12)^4}$$

$$= \text{Rs. } 317288$$

Compound all cash inflows to N i.e 15 years at 12%

$$55000\,(F/A, 12\%, 11) + 330000$$

$$= 55000 \times \frac{(1+0.12)^{11} - 1}{0.12} + 330000$$

$$= \text{Rs. } 1466003$$

Equivalence,

$$317288\,(F/P, i\%, 15) = 1466003$$

$i\% = 10.74$ i.e. ERR = 10.74% < MARR

Therefore, investment is not good.

3.5 Public Sector Economic Analysis (Benefit Cost Ratio Method)

When evaluating public project, taxes and pay off periods are considered less significant and indirect effects, interest rate, over counting, tolls, fees user charges etc are more significant. In the evaluation of such projects we often use benefit cost analysis (BCA). BCA also provides information about the relative economic efficiency of alternatives.

B/C Ratio (BCR)

Benefit cost ratio is defined as the ratio of equivalent worth of user's benefits to the equivalent worth of sponsor's cost.

Types of B/C ratio

1. **Conventional B/C ratio**

   It is the ratio of gross benefit to costs and expressed as

   a) **With PW formulation**

$$BCR = \frac{PW(B)}{PW(I) - PW(S) + PW(O\&M)}$$

   b) **With FW formulation**

$$BCR = \frac{FW(B)}{FW(I) - FW(S) + FW(O\&M)}$$

   c) **With AW formulation**

$$BCR = \frac{AW(B)}{AW(I) - AW(S) + AW(O\&M)}$$

2. **Modified B/C ratio**

   It is the ratio of net benefits to the costs and expressed as,

   a) **With PW formulation**

$$BCR = \frac{PW(B) - PW(O\&M)}{PW(I) - PW(S)}$$

   b) **With FW formulation**

$$BCR = \frac{FW(B) - FW(O\&M)}{FW(I) - FW(S)}$$

   c) **With AW formulation,**

$$BCR = \frac{AW(B) - AW(O\&M)}{AW(I) - AW(S)}$$

Where, B = benefit

I = investment

S = salvage

O & M = Operation and maintenance

Decision Rule

IF BCR > 1, Accept the project

If BCR = 1, remain indifferent

If BCR < 1, reject the project.

Project to be feasible if

Benefit (B) > cost (C)

OR, BCR > 1

Example:

Find both types of B/C ratio using PW, FW and AW method.

Initial investment = Rs. 2000

Revenue/year = Rs. 1000

Expenses/year = Rs. 440

Salvage value = Rs. 400

Useful life = 5 years

MARR = 8%

Solution:

A) Using PW method

$$PW(B) = 1000\,(P/A, 8\%, 5)$$

$$= 1000\frac{(1.08^5-1)}{1.08^5 \times 0.08}$$

$$= Rs.\ 3992.7$$

$$PW(S) = 4000\,(P/F, 8\%, 5)$$

$$= \frac{400}{1.08^5}$$

$$= Rs.\ 272.23$$

$$PW(O\&M) = 440\,(P/A, 8\%, 5)$$

$$= 440\frac{(1.08^5-1)}{1.08^5 \times 0.08} = Rs.\ 1756.79$$

$$\text{Conventional B/C ratio} = \frac{PW(B)}{PW(I) - PW(S) + PW(O\&M)}$$

$$= \frac{3992.7}{2000 - 272.23 + 1756.79} = 1.146 > 1, \text{Ok}$$

$$\text{Modified B/C ratio} = \frac{PW(B) - PW(O\&M)}{PW(I) - PW(S)}$$

$$= \frac{3992.7 - 1756.79}{2000 - 272.23}$$

$$= 1.294\ (\text{Ok})$$

2. Using AW method

$$\text{Conventional B/C ratio} = \frac{AW(B)}{CR + AW(O\&M)}$$

$$CR = I - S$$

$$CR = 2000\,(A/P, 8\%, 5) - 400\,(A/F, 8\%, 5)$$

$$= 2000 \times \frac{1.08^5 \times 0.08}{(1.08^5 - 1)} - 400 \times \frac{0.08}{(1.08^5 - 1)}$$

$$= 432.7$$

$$\text{Conventional B/C ratio} = \frac{1000}{(432.7 + 440)} = 1.146 > 1\ (\text{ok})$$

$$\text{Modified B/C ratio} = \frac{AW(B) - AW(O\&M)}{CR}$$

$$= \frac{1000 - 440}{432.7} = 1.294 > 1\ (\text{ok})$$

3. Using FW method

$$FW(B) = 1000\,(F/A, 8\%, 5) = 1000\frac{(1.08^5-1)}{0.08} = 5866.6$$

$$FW(S) = 400$$

$$FW(O\&M) = 440\,(F/A, 8\%, 5)$$

$$= 440\frac{(1.08^5-1)}{0.08}$$

$$= 2581.3$$

$$FW(I) = 2000\,(F/P, 8\%, 5)$$

$$= 2000 \times 1.08^5$$

$$= 2938.65$$

$$\text{Conventional B/C ratio} = \frac{FW(B)}{FW(I) - FW(S) + FW(O\&M)}$$

$$= \frac{5866.6}{2938.65 - 400 + 2581.3}$$

$$= 1.146 > 1\ (\text{ok})$$

$$\text{Modified B/C Ratio} = \frac{FW(B) - FW(O\&M)}{FW(I) - FW(S)}$$

$$= \frac{5866.6 - 2581.3}{2938.65 - 400} = 1.294 > 1\ (\text{ok})$$

3.6 Introduction to Life Cycle Costing

Because 80% of the total life-cycle cost of a system occurs after the system has entered service, the best long-term system acquisition and support decisions are based on a full understanding of the total cost of acquiring, operating, and supporting the system.

Sum of all recurring and one time (non-recurring) cost over the full life span or a specified period of a good, service, structure or system. In includes purchase price, installation cost, operation cost, maintenance and upgrade cost, and remaining (residual or salvage) value at the end of ownership or its useful life. Generally, life cycle cost estimates may be categorized into a simplified format for the major phases of acquisition and operation and their respective stages.

- **Acquisition phase:** All activities prior to the delivery of products and services.
- **Requirements definition stage :** Determination of users & customer needs
- **Preliminary design stage :** Includes feasibility study, conceptual and early stage plans
- **Detailed design stage :** Include detailed plans for resources, capital, human, facilities, information system, marketing etc.
- **Operation phase :** All activities are functioning product and services are available
- **Construction and implementation stage:** Includes purchase, construction and implementation of system.
- **Usage stage:** Generate products or services
- **phase out and disposal stage:** removal/recycling of old system

Characteristics of Life Cycle Costing:

a. Product life cycle costing involves tracing of costs and revenues of a product over several calendar periods throughout its life cycle.

b. Product life cycle costing traces research and design and development costs and total magnitude of these costs for each individual product and compared with product revenue.

c. Each phase of the product life-cycle poses different threats and opportunities that may require different strategic actions.

d. Product life cycle may be extended by finding new uses or users or by increasing the consumption of the present users.

Stages of Product Life Cycle Costing:

Following are the main stages of Product Life Cycle:

(i) **Market Research:** It will establish what product the customer wants, how much he is prepared to pay for it and how much he will buy.

(ii) **Specification:** It will give details such as required life, maximum permissible maintenance costs, manufacturing costs, required delivery date, expected performance of the product.

(iii) **Design:** Proper drawings and process schedules are to be defined.

(iv) **Prototype Manufacture:** From the drawings a small quantity of the product will be manufactured. These prototypes will be used to develop the product.

(v) **Development:** Testing and changing to meet requirements after the initial run. This period of testing and changing is development. When a product is made for the first time, it rarely meets the requirements of the specification and changes have to be made until it meets the requirements.

(vi) **Tooling:** Tooling up for production can mean building a production line; building jigs, buying the necessary tools and equipment's requiring a very large initial investment.

(vii) **Manufacture:** The manufacture of a product involves the purchase of raw materials and components, the use of labour and manufacturing expenses to make the product.

(viii) Selling

(ix) Distribution

(x) Product support

(xi) **Decommissioning:** When a manufacturing product comes to an end, the plant used to build the product must be sold or scrapped.

Benefits of Product Life Cycle Costing:

Following are the main benefits of product life cycle costing:

(i) It results in earlier action to generate revenue or lower costs than otherwise might be considered. There are a number of factors that need to be managed in order to maximize return in a product.

(ii) Better decision should follow from a more accurate and realistic assessment of revenues and costs within a particular life cycle stage.

(iii) It can promote long term rewarding in contrast to short term rewarding.

(iv) It provides an overall framework for considering total incremental costs over the entire span of a product.

Life Cycle Costing Process: Life cycle costing is a three-staged process. The first stage is life cost planning stage which includes planning LCC Analysis, Selecting and Developing LCC Model, applying LCC Model and finally recording and reviewing the LCC Results. The Second Stage is Life Cost Analysis Preparation Stage followed by third stage Implementation and Monitoring Life Cost Analysis.

The three stages are:

LCC Analysis is a multi-disciplinary activity. An analyst, involved in life cycle costing, should be fully familiar with unique cost elements involved in the life cycle of asset, sources of cost data to be collected and financial principles to be applied.

He should also have clear understanding of methods of assessing the uncertainties associated with cost estimation. Number of iteration may be required to perform to finally achieve the result. All these iterations should be documented in detail to facilitate the interpretations of final result.

Stage 1: LCC Analysis Planning: The Life Cycle Costing process begins with development of a plan, which addresses the purpose, and scope of the analysis.

The plan should:

i. Define the analysis objectives in terms of outputs required to assist a management decision.

Typical objectives are:

a. Determination of the LCC for an asset in order to assist planning, contracting, budgeting or similar needs.

b. Evaluation of the impact of alternative courses of action on the LCC of an asset (such as design approaches, asset acquisition, support policies or alternative technologies).

c. Identification of cost elements which act as cost drives for the LCC of an asset in order to focus design, development, acquisition or asset support efforts.

ii. Make the detailed schedule with regard to planning of time period for each phase, the operating, technical and maintenance support required for the asset.

iii. Identify any underlying conditions, assumptions, limitations and constraints (such as minimum asset performance, availability requirements or maximum capital cost limitations) that might restrict the range of acceptable options to be evaluated. Identify alternative courses of action to be evaluated.

iv. Identify alternative courses of action to be evaluated. The list of proposed alternatives may be refined as new options are identified or as existing options are found to violate the problem constraints.

v. Provide an estimate of resources required and a reporting schedule for the analysis to ensure that the LCC results will be available to support the decision-making process for which they are required.

Next step in LCC Analysis planning is the selection or development of an LCC model that will satisfy the objectives of the analysis. LCC Model is basically an accounting structure which enables the estimation of an asset components cost.

Stage 2: Life Cost Analysis Preparation: The Life Cost Analysis is essentially a tool, which can be used to control and manage the ongoing costs of an asset or part thereof. It is based on the LCC Model developed and applied during the Life Cost Planning phase with one important difference: it uses data on real costs.

The preparation of the Life Cost Analysis involves review and development of the LCC Model as a **"real-time"** or actual cost control mechanism. Estimates of capital costs will be replaced by the actual prices paid. Changes may also be required to the cost breakdown structure and cost elements to reflect the asset components to be monitored and the level of detail required.

Targets are set for the operating costs and their frequency of occurrence based initially on the estimates used in the Life Cost Planning phase. However, these targets may change with time as more accurate data is obtained, from the actual asset operating costs or from the operating cost of similar other asset.

Stage 3 : Implementing and Monitoring: Implementation of the Life Cost Analysis involves the continuous monitoring of the actual performance of an asset during its operation and maintenance to identify areas in which cost savings may be made and to provide feedback for future life cost planning activities.

For example, it may be better to replace an expensive building component with a more efficient solution prior to the end of its useful life than to continue with a poor initial decision.

3.7 Introduction to Financial and Economical Analysis

Financial and economic analyses have similar features. Both estimate the net-benefits of a project investment based on the difference between the with-project and the without-project situations.

The basic difference between them is that:

- the financial analysis compares benefits and costs to the enterprise, while

- the economic analysis compares the benefits and costs to the whole economy

Methodology

Economic analysis is concerned with the true value a project holds for the society as a whole. It subsumes all members of society, and measures the project's positive and negative impacts. In addition, economic analysis would also cover costs and benefits of goods and services that are not sold in the market and therefore have no market price.

There are two more significant differences between financial and economic analysis:

- While **financial analysis uses market prices** to check the balance of investment and the sustainability of a project,
- **Economic analysis uses economic prices** that are converted from the market price by **excluding tax, profit, subsidy**, etc. to measure the legitimacy of using national resources to certain projects.
- Financial and economic analyses also differ in their treatment of external effects (benefits and costs), such as favorable effects on health. Economic analysis attempts to value such externalities in order to reflect the true cost and value to the society. The inclusion of externalities raises difficult questions of their identification and measurement in terms of money.

Underlining the integrated perspective of the IFM approach, cost-benefit analysis in the present context, is meant to be a form of economic analysis.

However, it has to be noted that economic and financial analysis are also complementary. For a project to be economically viable, it must be financially sustainable. If a project is not financially sustainable, there will be no adequate funds to properly operate, maintain and replace assets. It has sometimes been suggested that financial viability should not be made a concern because as long as a project is economically sound, it can be supported through government subsidies.

Example: What are the differences between Financial Analysis (FA) and Economic Analysis (EA) ?

Ans: The basic principles for carrying out FA and EA are the same and both are required for project screening and selection. However, there is a difference in approach; FA deals with the cost and benefit flows from the point of view of the individual, firm or institution, while EA deals with the costs and benefits to society. EA takes a broader view of costs and benefits, and the methods of analysis differ in a number of important respects. An enterprise is interested in financial profitability and the sustainability of that profit, while society is concerned with wider objectives, such as food security, poverty alleviation, and net benefits to society as a whole. The main differences between FA and EA are that EA: (i) attempts to quantify "externalities" (i.e. negative or positive effects on specific groups in society without the project entity incurring a corresponding monetary cost or enjoying a monetary benefit. This includes both environmental and social impacts resulting from the project); (ii) removes transfer payments (i.e. subsidies and taxes); and (iii) makes use of "shadow prices" that might differ from the "market prices", in order to eliminate market distortions and reflect the effective opportunity costs for the economy, thus achieving a proper valuation of economic costs and benefits from the perspective of the economy as a whole.

Financial analysis

Capital requirement, source of fund, projected cash flows profitability and project's capacity to meet financial obligations are the area focused by financial analysis.

Breakeven analysis and sensitivity analysis is done to test the effect of changes in variables such as cost, price, time etc.

Economical analysis

It analyze the economic viability of the project, for private sector-profitability is the major concern and satisfactory rate of return is needed and for public sector profitability is in terms of contribution to national economy is considered.

IRR > MARR, BCR > 1, NPV > 0 are the decision criteria.

Economic Analysis	Financial Analysis
1. It is done by government project.	1. It is done by private project.
2. Evaluates the monitory and non-monitory benefits also convert social benefits.	2. Evaluates monitory benefits only.
3. Consumer oriented	3. Investor oriented.
4. Profit is never a goal.	4. Profit is most.
5. B/C ratio analysis is most used.	5. ROR and PW are most used.

Old Question Solution

1. **From the following cash flow**

EOY	0	1	2	3	4	5
Cash flow	-3000	800	1000	1100	1210	1464

Calculate both type of payback period. MARR = 10% **[2069 Bhadra]**

Solution:

Year	Cash flow	Cumulative Flow (CF)	PW of cash flow	CF
0	-3000	-3000	-3000	-3000
1	800	-2200	$800/1.1 = 727.27$	-2272.73
2	1000	-1200	$1000/1.1^2 = 826.44$	-1446.29
3	1100	-100	$1100/1.1^3 = 826.44$	-619.85
4	1210	1110	$1210/1.1^4 = 826.44$	206.85
5	1464	2574	$1464/1.1^5 = 909.028$	1033.03

a) Simple payback period

Year	CF
3	-100
4	1110
? (3.083)	0

By interpolation

Hence simple payback period = 3.083 years

b) Discounted payback period

CF	Year
-619.85	3
206.85	4
0	? (3.75)

Hence discounted payback period = 3.75 years

2. **Equipment costs 250000 and has salvage value of 50000 at the end of its expected life 5 years. Annual expenses will be 40000. It will produce revenue of 120000 per year.**

MARR = 20% = ∈

i) Evaluate IRR using AW formulation

ii) Evaluate both type of B/C ratio with FW formulation

iii) Find ERR **[2069 Bhadra]**

Solution:

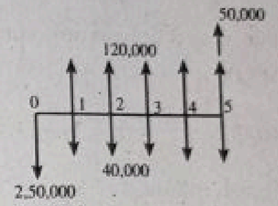


Fig. Cash-Flow Diagram

a) For IRR

Let i% be the IRR, then

AW (i%) = 0

$-250000 (A/P, i\%, 5) + 120000 - 40000 + 50000 (A/F, i\%, 5) = 0$

or, $-250000 \times \frac{i \times (1+i)^5}{[(1+i)^5 - 1]} + 80000 + 50000 \times \frac{i}{[(1+i)^5 - 1]} = 0$

On solving, we get

$i = 0.2157 = 21.57\% > \text{MARR } (20\%)$ (Justified)

b) BCR

$FW (I) = 250000 (F/P, 20\%, 5)$

$= 250000 \times (1 + 0.2)^5 = 622080$

$FW (O \& M) = 40000 (F/A, 20\%, 5)$

$= 40000 \times \frac{((1+0.2)^5 - 1)}{0.2} = 297,664$

$FW (B) = 120000 (F/A, 20\%, 5)$

$= 120000 \times \frac{[(1+0.2)^5 - 1]}{0.2} = 892992$

Now, Convention $B/C = \frac{FW(B)}{FW (I) - FW(S) + FW (O \& M)}$

$= \frac{892992}{622080 - 50000 + 297664} = 1.026 > 1$ (justified)

$$\text{Modified B/C} = \frac{FW(B) - FW(O\ \&\ M)}{FW(I) - FW(S)}$$

$$= \frac{892997 - 297664}{622080 - 50000}$$

$$= 1.0406 > 1 \text{ (justified)}$$

C) ERR

a) Discounting all cash out flow at time zero

$$= -250{,}000 - 40000\,(P/A, 20\%, 5)$$

$$= -250000 - 40000 \times \frac{1.2^5 - 1}{0.2 \times 1.2^5} = 369624.48$$

b) Discounting all cash inflows to 5 years (N)

$$= 120000 \times (F/A, 20\%, 5) + 50000$$

$$= 120000 \times \frac{1.2^5 - 1}{0.2} + 50000 = 942992.00$$

Equating with ERR i%

$$369624.48\,(1 + i)^5 = 942992.00$$

On solving, we get

$$i = ERR = 20.6\%$$

3. **Define equivalent worth and rate of return method. How much rupees should you deposit now in a bank account that gives 8% interest per year if you wish to draw Rs. 10000 per month for 10 years?** [2070 Magh]

Solution:

Equivalent worth:

The equivalent value of all the cash in and out flows of an organization at year 0 or annual basis or at year N is called the equivalent worth. Types of equivalent worth.

1. Present worth method
2. Future worth method
3. Annual worth method

Rate of return

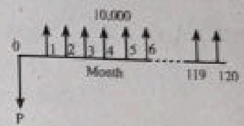
Rate of return is the break-even interest rate, i which equates the present worth of a project's. Cash outflows to the present worth of its cash inflows.

$i_{eff} = 8\%$ per year

If $i_{monthly}$ be the interest rate for month then

$$i_{monthly} = (1 + i_{eff})^{1/12} - 1$$

$$= (1 + 0.08)^{1/12} - 1 = 0.643\%$$



Using NPW (i_m %) = 0

$$-P + 10000\,(P/A, 0.643\%, 120) = 0$$

$$P = 10000 \times \frac{1.00643^{120} - 1}{0.00643 \times 1.00643^{120}}$$

$$= \text{Rs. } 8{,}34{,}500.6$$

∴ We have to deposit a sum of Rs. 834500.60 at the beginning to draw Rs. 10000 each for next 10 years.

4. **What is the different between financial and economic analysis? Determine both type of B/C ratio from the following cash flow.**

Initial investment	=	300000
Annual revenue	=	85000
Annual cost	=	15000
Salvage value	=	20% of initial investment
Useful life	=	6 years
MARR	=	10%

[2070 Magh]

Solution:

Economic Analysis	Financial Analysis
1. It is done by government project.	1. It is done by private project.
2. Evaluates the monitory and non-monitory benefits also convert social benefits.	2. Evaluates monitory benefits only.
3. Consumer oriented	3. Investor oriented.
4. Profit is never a goal.	4. Profit is most.
5. B/C ratio analysis is most used.	5. ROR and PW are most used.

Using PW approach,

PW (I) = 300000

(S) = 0.2×300000

= 60000

PW (S) = 60000 (P/F, 10%, 6)

$= \frac{60000}{1.1^6}$

= 33868.44

PW (O & M) = 15000 (P/A, 10%, 6)

$= 15000 \times \frac{[(1+0.1)^6 - 1]}{0.1 \times (1+0.1)^6}$

= 65328.91

PW (B) = 85000 (P/A, 10%, 6)

$= 85000 \times \frac{(1.1^6 - 1)}{0.1 \times 1.1^6}$

= 370192.16

Now,

$$\text{B/C conventional} = \frac{\text{PW (B)}}{\text{PW(I)} - \text{PW (S)} + \text{PW(O \& M)}}$$

$$= \frac{370197.16}{300000 - 33868.44 + 65328.391}$$

= 1.12 > 1 (Justified)

$$\text{Modified B/C} = \frac{\text{PW (B)} - \text{PW (O \& M)}}{\text{PW (I)} - \text{PW (S)}}$$

$$= \frac{370197.16 - 65328.91}{300000 - 33868.44} = 1.145 > 1 \text{ (Justified)}$$

5. **Compute IRR by using trial and error process of the following projects. Determine also investment decision.**

Initial investment = 25000

Annual income = 8000

Salvage value = 5000

Useful life = 5 year

MARR = 20%

[2070 Magh]

Solution:

Using PW method,

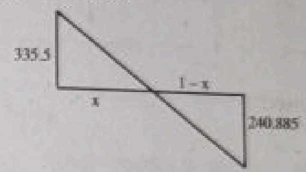
PW (i %) = 0

PW (i %) = -25000 + 8000 (P/A, i%, 5) + 5000 (P/F, i%, 5000)

$$= -25000 + 8000 \frac{(1+i)^5 - 1}{i(1+i)^5} + \frac{5000}{(1+i)^5} \quad \text{.......... (1)}$$

Put, i = 21%, PW (21%) = 335.50

I = 22%, PW (22%) = -240.885



$$\frac{1-x}{x} = \frac{240.885}{335.5}$$

Solving x = 0.582

At i = 21 + 0.582 = 21.582, PW (21.582) = -2.59

∴ i lies between 21% and 21.582%

Interpolation

21% 335.5

21.582% -2.59

(21.577)? 0

Now, i = 21.577 at equation (1)

PW (21.577) = 0

∴ IRR = 21.577 > MARR (20%)

∴ Economically justified for investment

6. **Initial investment = Rs. 100000**

Salvage value = 0

Annual O & M cost = Rs. 20000

Useful life = 5 years

Annual benefit = 60000 at the end of first year;

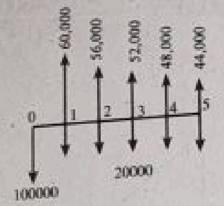
Thereafter decreases by 4000 each year for the remaining years.

a) **Draw U/B diagram**

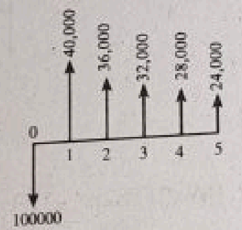
b) Evaluate conventional BCR using PW formulation. Take salvage value = 10000

c) Evaluate discounted payback period. Take standard (cut off) payback period = 3 years. **[2070 Bhadra]**

Solution: UIB (Unrecovered Investment Balance) Diagram:



This can also represented by



Let i% be the internal rate of interest then,

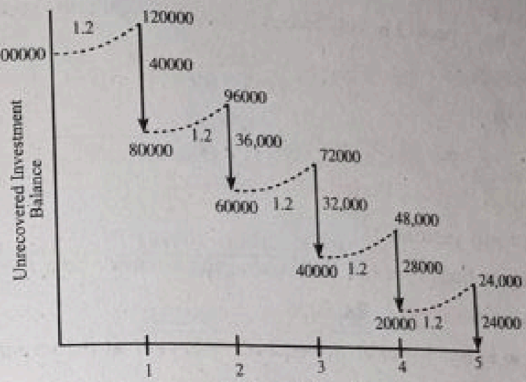
PW (i%) = 0

or, $-100000 + \frac{40000}{(1+i)} + \frac{36000}{(1+i)^2} + \frac{32000}{(1+i)^3} + \frac{28000}{(1+i)^4} + \frac{24000}{(1+i)^5}$

On solving,

i = 0.2 = 20% = MARR. Remain indifferent.

Year	Unpaid balance at beginning year (Rs.)	Return on unpaid Balance (Rs.)	Payment received	Unpaid balance at end of year (Rs.)
0	-100000	0	0	- 100000
1	-100000	100000 × 0.2 = -20000	40000	- 80000
2	-80000	-80000 × 0.2 = -16000	36000	- 60000
3	-60000	-12000	32000	- 40000
4	-40000	-8000	28000	- 20000
5	-20000	-4000	24000	0



Investment balance diagram

b) B/C ratio, MARR = 12%

$$PW(B) = \frac{60000}{1.12} + \frac{56000}{1.12^2} + \frac{52000}{1.12^3} + \frac{48000}{1.12^4} + \frac{44000}{1.12^5}$$

$$= 190628.5$$

$$PW(S) = \frac{10000}{1.12^5} = 5674.26$$

$$PW(O \& M) = 20000 \times \frac{(1.15^6 - 1)}{0.12 \times 1.12^5} = 72095.52$$

$$\text{Conventional B/C ratio} = \frac{PW(B)}{PW(I) + PW(O \& M) - PW(S)}$$

$$= \frac{190698.5}{100000 + 72096.5 - 5674.26} = 1.145 > 1$$

Economically justified

C) Discounted payback period

Year	Cash flow	PW of CF	Cumulative CF
0	-100000	-100000	-100000
1	40000	40000/1.12 = 35714	-64285.2
2	36000	$36000/1.12^2$ = 28698.9	-35586.2
3	32000	22776.9	-12809.25
4	28000	17794.5	4985.50
5	24000	13618.24	18603.44

∴ The payback period is in between 3 and 4 years.

-12809.25	3
4985.50	4
0	? (3.72) years

∴ Discounted Payback Period = 3.72 years ≯ 3 years

∴ Economically justified

7. **Calculate both types of BCR using FW formulation**

   Initial investment is Rs. 50000

   Income is Rs. 10000 at the end of first year and increasing by 10% per year

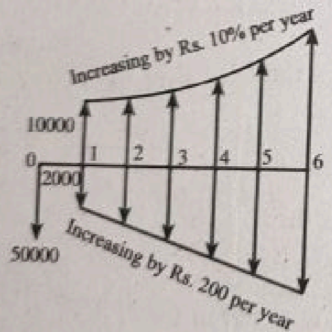
   Annual expenditure is Rs. 2000 at the end of first year and increasing by Rs. 200 per year

   Useful life is 6 years

   MARR = 15% **[2071 Magh]**

Solution:

$$FW(I) = 50000\,(F/P, 15\%, 6)$$
$$= 50000 \times 1.15^6 = 115653.038$$



$$FW(O\&M) = 2000\,(F/A, 15\%\, 6) + \frac{G}{i}(F/G, 15\%, 6) - \frac{NG}{i}$$
$$= 2000 \times \frac{(1.15^6 - 1)}{0.15} + \frac{200}{0.15}\frac{(1.15^6 - 1)}{0.15} - \frac{6 \times 200}{0.15}$$
$$= 21179.128$$
$$i = 15\%, g = 10,\%$$

$$FW(B) = A\frac{((1+i)^N - (1+g)^N)}{i - g} = 10000 \times \frac{(1.15^6 - 1.1^6)}{0.15 - 0.1} = 108299.95$$

∴ From conventional BCR method

$$FW(B) = \frac{FW(B)}{FW(I) - FW(S) + FW(O\&M)}$$
$$= \frac{108299.953}{115653 - 20000 + 21179.128} = 0.926 < 1$$

Not justified.

$$\text{Modified BCR} = \frac{FW(B) - FW(O\&M)}{FW(I) - FW(S)}$$
$$= \frac{108299.953 - 21179.128}{115653 - 20000} = 0.910 < 1$$

Not justified

8. **Define IRR. Find d IRR and ERR of the following project.**

   MARR = ∈ = 15%

Year	0	1	2	3	4
Cash flow	-50	-10	30	40	50

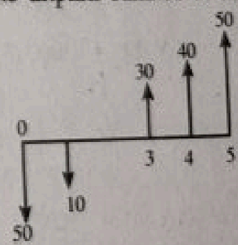
[2071 Bhadra]

Solution:

IRR is the breakeven interest rate 'i' which equates the present worth of a project's cash out flows to the present worth of its cash inflows.

In another way,

IRR is defined as the interest rate earned on the unpaid balance of an installment loan.



$$PW(i\%) = 0$$

$$-50 - \frac{10}{(1+i)} + \frac{30}{(1+i)^3} + \frac{40}{(1+i)^4} + \frac{50}{(1+i)^5} = 0$$

Solving,

$$i = 16.21\% > MARR\ (15\%)$$

Discounting all the cash outflow at present with ∈ = 15%

$$= -5 - \frac{100}{1.15} = -58.695$$

Discounting all the cash inflow at the end of 5th year

$$30 \times 1.15^2 + 40 \times 1.15 + 50 = 135.675$$

Establishing equivalence

$58.695 \times (1 + i)^5 = 135.68$, solving i = 0.1824

i = 18.24% > MARR

∴ ERR = 18.24%

8.

	Machine A
Initial investment	Rs. 6000
Annual benefits	Rs. 3000
O & M cost	Rs. 1000
Salvage value	Rs. 1500
MARR	10%

a) **Evaluate both type of BCR (FW formulation). Take useful life = 10 years.**

b) **Evaluate both type of payback period. If useful life = 5 years. (Take standard payback period = 3 years).**

c) **Explain the factors affecting determination of MARR.**

[2071 Bhadra]

Solution:

FW (I) = 6000 (F/P, 10%, 10)

$= 6000 \times 1.1^{10} = 15562.45$

FW (B) = 3000 (F/A, 10%, 10)

$= 3000 \times \frac{(1.1^{10} - 1)}{0.1}$

= 47812.27

FW (O & M) = 1000 (F/A, 10%, 10)

$= 1000 \times \frac{1.1^{10} - 1}{0.1}$

= 15937.42

FW(S) = 1500

Conventional B/C ratio $= \frac{FW(B)}{FW(I) - FW(S) + FW(O\&M)}$

$= \frac{47812.27}{15562.45 - 1500 + 15937.42}$

= 1.59 > 1, justified

Modified B/C ratio $= \frac{FW(B) - FW(O\&M)}{FW(I) - FW(S)}$

$= \frac{47812.27 - 15937.42}{15562.45 - 1500} = 2.26 > 1$

b) Payback period

Year	CF	Cumulative CF	PW of CF	Cumulative CF (Discounted)
0	-6000	-6000	-6000	-6000
1	3000 – 1000 = 2000	-4000	1818.18	-4181.82
2	2000	-2000	1652.89	-2528.93
3	2000	0	1502.63	-1026.33
4	2000	2000	1366.026	339.726
5	2000 + 1500 = 3500	55000	2173.22	2512.946

i) Simple payback period = 3 years = standard payback period.

Decision → Remain indifferent

ii) Discounted payback period

–1026.33 3

339.726 4

0 ? (3.75)

Discounted payback period = 3.75 > 3 years

Economically not justified.

iii) *Refer at 3.1*

9. **Calculate IRR from the following cash flow and draw investment balance diagram.**

Year	0	1	2	3	4	5
Cash flow	-800	250	300	400	-150	600

[2072 Ashwin]

Solution:

PW(i%) = 0

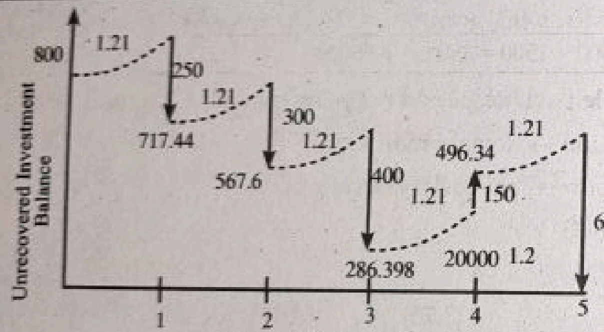
$-800 + \frac{250}{(1+i)} + \frac{300}{(1+i)^2} + \frac{400}{(1+i)^3} - \frac{150}{(1+i)^4} + \frac{600}{(1+i)^5} = 0$

Solving, we get, i = 0.2093

i = 20.93%

Unrecovered project balance calculation

Year	Unpaid balance at beginning year (Rs.)	Return on unpaid Balance (Rs.)	Payment received	Unpaid balance at end of year
0	0	0	-800	-800
1	-800	-167.44 (800 × 0.2093)	250	-717.44
2	-717.44	-150.16 (717.44 × 0.2093)	300	-567.6
3	-567.6	-118.79	400	-286.398
4	-286.398	-59.94	-150	-496.34
5	-496.34	-103.8	600	0.14 ≈ 0



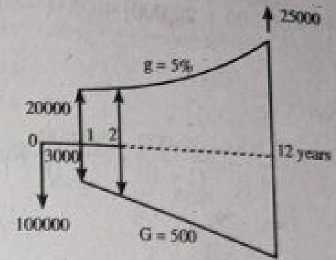
Investment balance diagram

10. Calculate both types of BCR of a project with following details: MARR = 12%

Initial investment	Annual income	Annual cost	Useful life	Salvage
Rs. 100000	Rs. 20000 at the end of first year and increased by 5% per year	Rs. 3000 at the end of first year and increased by Rs. 500 per year	12 years	25000

[2072 Ashwin]

Solution:



Using a FW formulation:

$$FW(I) = 100000\,(F/A, 12\%, 12)$$
$$= 100000 \times 1.12^{12}$$
$$= 389597.599$$

$$FW(B) = A\frac{(1+i)^N - (1+g)^N}{i-g}$$
$$= 20000\frac{[(1+0.12)^{12} - (1+0.05)^{12}]}{0.12-0.05} = 600034.19$$

$$FW(O\&M) = 3000\,(F/A, 12\%, 12) + \frac{G}{i}(F/G, i\%, N) - \frac{NG}{i}$$
$$= 3000 \times \frac{(1.12^{12}-1)}{0.12} + \frac{500}{0.12} \times \frac{1.12^{12}-1}{0.12} - \frac{12 \times 500}{0.12}$$
$$= 122954.12$$

$$FW(S) = 25000$$

$$\text{Conventional B/C ratio} = \frac{FW(B)}{FW(I) - FW(S) + FW(O\&M)}$$
$$= \frac{600034.19}{389597.599 - 25000 + 122954.12} = 1.23 > 0$$

Economically justified.

$$\text{Modified B/C ratio} = \frac{FW(B) - FW(O\&M)}{FW(I) - FW(S)}$$
$$= \frac{600034.19 - 122954.12}{389597.599 - 25000} = 1.3 > 1 \text{ Justified}$$

11. Use IRR method to evaluate following project when MARR is 15%. Make also unrecovered balance graph. [2073 Bhadra]

EOY	0	1	2	3	4	5
Cash Flow	- 60,000	20,000	40,000	-40,000	50,000	70,000

[2073 Bhadra]

Solution: PW(i%) = 0

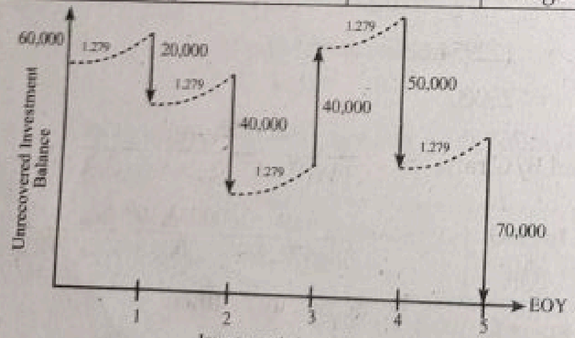
$$-60{,}000+\frac{20{,}000}{(1+i)}+\frac{40{,}000}{(1+i)^2}-\frac{40{,}000}{(1+i)^3}+\frac{50{,}000}{(1+i)^4}+\frac{70{,}000}{(1+i)^5}=0$$

Solving, we get, i = 0.2797

I = 27.97%

Unrecovered Project Balance Calculation

Year	Unpaid balance at beginning year (Rs)	Return on Unpaid Balance (Rs.)	Payment Received	Unpaid Balance at end of year
0	0	0	- 60,000	-60,000
1	- 60,000	- 16782 (60,000 × 0.2797)	20,000	- 56,782 (-60,000 - 16782 + 20,000)
2	- 56782	- 15881.93	40,000	- 32663.93
3	- 32663.93	- 9136.09	- 40,000	- 81800
4	- 81800	- 22879.46	50,000	- 54679.46
5	- 54679.46	- 15293.84	70,000	26.64 ≈ 0 (With respect to high amount)



Investment balance diagram

12. **Your college is considering to purchase a vehicle of Rs. 3,00,000 expecting salvage value Rs. 50,000 at the end of 10th year. The use of vehicle saves Rs. 80,000 per year. When it needs Rs. 20,000 operating cost for each year. Find**

(i) **Both type of B/C ratio of FW formulation**

(ii) **Both types of payback period.**

[2073 Bhadra]

Solution:

Given, Initial investment = 3,00,000

Annual benefits = 80,000

O & M cost = 20,000

Salvage value = 50,000

MARR = Assume 10%

Useful time = 10 years.

1) **B/C ratio method**

F W (I) = 3,00,000 (F/P, 10%, 10)

= 778122.738

FW (B) = 80,000 (F/A, 10%10)

$$= 80{,}000 * \frac{(1.1^{10}-1)}{0.1}$$

= 1274993.968

FW (O & M) = 20,000 (F/A, 10%, 10)

$$= 20{,}000 * \frac{1.1^{10}-1}{0.1}$$

= 318748.492

FW(S) = 50,000

$$\text{Conventional B/C ratio} = \frac{FW(B)}{FW(I) - FW(S) + FW\ (O\ \&\ M)}$$

$$= \frac{1274993.968}{778122.738 - 50{,}000 + 318748.492}$$

= 1.217 > 1, Justified

$$\text{Modified B/C ratio} = \frac{FW(B) - FW\ (O\ \&\ M)}{FW(I) - FW\ (S)}$$

$$= \frac{1274993.968 - 318748.492}{778122.738 - 50{,}000}$$

= 1.31 > 1 Justified

2) **Payback Period**

EOY	Cash Flow (CF)	Cumulative CF	PW of CF	Cumulative CF (Discounted)
0	- 3,00,000	- 3,00,000	- 3,00,000	-3,00,000
1	80,000 - 20,000 = 60,000	- 24,00,000	60,000/1.1 = 54545.45	- 245454.55

2	60,000	- 180000	$60{,}000/1.1^2$ = 49586.776	- 195867.77
3	60,000	- 120000	45078.88	- 150788.89
4	60,000	- 60000	40980.8	- 109808.09
5	60,000	0	37255.27	- 72552.824
6	60,000	60000	33868.44	- 38684.38
7	60,000	120000	30789.48	- 7894.904
8	60,000	18000	27990.44	20095.53
9	60,000	240000	25445.85	45541.386
10	50,000 + 60,000	300000	$110000/1.1^{10}$ = 42409.76	87951.146

i) **Simple Payback Period = 5 years (zero CF)**

ii) **Discount Payback Period,**

CF	Year
- 7894.904	7
20095.53	8
0	? (7.28)

Hence discounted payback period = 7.28 years

13. **Distinguish between financial and economic analysis.** **2073 Bhadra]**

Solution: *Refer [2070 Magh Q.No. 2]*

14. **Initial investment of a project is Rs. 1,00,000 having useful life is equal to 5 years with no salvage value. Annual operation and maintenance cost is Rs. 25,000. Annual revenue at the end of first year is Rs. 70,000 and decrease by Rs. 5,000 each year for the remaining years.**
    i) **Draw u/B diagram**
    ii) **Evaluate modified BCR using PW formulation. Take salvage value**
    iii) **Evaluate discounted payback period. Take standard (cut off)**

    Payback period 3 years. **[2073 Magh]**

Solution: *Refer "2070 Bhadra"*

15. **Explain any two drawbacks of IRR with example. Differentiate between economic analysis and financial analysis. [2074 Bhadra]**

Solution: *Refer For IRR drawbacks*

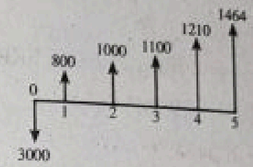
Refer (2070 Magh) Q.N. 2 for difference.

16. **Evaluate the project by using AW formulation of the project at i = 12%**

EOY	0	1	2	3	4	5
Cashflow	-3000	800	1000	1100	1210	1464

[2074 Bhadra]

Solution: From given data, i = 12% = 0.12



AW (i%) = R – E – (RC%)

Converting all cashflow into future worth FW.

$$FW(12\%) = -3000(1+0.12)^5 + 800(1+0.12)^4 + 1000(1+0.12)^3 + 1100(1+0.12)^2 + 1210(1+0.12)^1 + 1464$$
$$= -5287.02 + 1258.82 + 1404.93 + 1379.84 + 1365.2 + 1464$$
$$= 1575.77$$

Now, $FW = A\left[\frac{(1+i)^n - 1}{i}\right]$

$$1575.77 = A\left[\frac{(1+0.12)^5 - 1}{0.12}\right]$$

On solving, A = 107.86

i.e. AW (12%) > 0, economically justified.

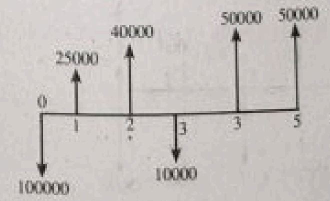
17. **Calculate the ERR of the following cash flow. MARR = 12%, reinvestment rate = 14%.**

EOY	0	1	2	3	4	5
Cashflow	-100,000	25,000	40,000	- 10,000	50,000	50,000

[2074 Bhadra]

Solution:

From given data MARR = 12% and reinvestment rate = 14%



Discount all cash out flow to time zero.

$$\text{Expenses (E)} = 100000 + \frac{10000}{(1+0.014)^3} = 106749.71$$

Compound all cash inflows to N i.e. 5 years at 14%,

$$\text{Revenues (R)} = 25000\,(1+0.14)^4 + 40000\,(1+0.14)^3 + 50000\,(1+0.14)^1$$
$$+ 50000 = 208485.76$$

Now, Equivalence outflow and inflow at rate ERR (i%)

$$106749.71\,(F/P, i\%, 5) = 208485.76$$

$$106749.71\,(1+i)^5 = 208485.76$$

On solving,

$$i = 0.1433$$

I.e. ERR = 14.33% > MARR

Therefore investment is good and accepted.

18. Make investment decision for the following project by using (i) IRR (ii) B/C (iii) Discounted payback methods. [2075 Baisakh]

Solution:

From Given data we have

Initial cost = – 400000

Annual revenue (A) = 160000 for 1st year

Decreases by i.e. Gradient (G) = – 10000

Annual expenses (E) = 40000 for 1st year

Increases by i.e. Gradient (G) = 5000

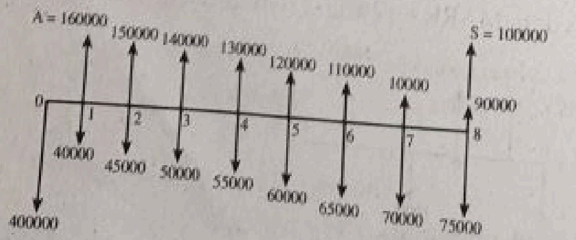
Salvage value (S) = Rs. 100000

N = 8 years

MARR = 9%

Now, (i) Using IRR

$$PW\,(i\%) = 0$$



Now,

BOY	0	1	2	3	4	5	6	7	8
Net cash flow	-400000	120000	105000	90000	75000	60000	45000	30000	115000

Using PW (i %) = 0

$$-400000 + \frac{120000}{(1+i)} + \frac{105000}{(1+i)^2} + \frac{90000}{(1+i)^3} + \frac{75000}{(1+i)^4} + \frac{60000}{(1+i)^5}$$
$$+ \frac{45000}{(1+i)^6} + \frac{30000}{(1+i)^7} + \frac{115000}{(1+i)^8} = 0$$

Taking 30,000 common from all term for easy calculation

$$-13.3 + \frac{4}{(1+i)} + \frac{3.5}{(1+i)^2} + \frac{3}{(1+i)^3} + \frac{2.5}{(1+i)^4} + \frac{2}{(1+i)^5} + \frac{1.5}{(1+i)^6}$$
$$+ \frac{1}{(1+i)^7} + \frac{3.83}{(1+i)^8} = 0$$

Assuming (1 + i) = x

Then solving by using calculator

$x = 1.1360$ $\quad \therefore i = 1.136 - 1 = 0.136 = 13.60\%$

I.e. IRR > MARR, economically feasible.

(ii) Using B/C method

For Linear gradient series A = 160000, G = – 10,000

Present worth of Benefit, PW (B) = A (P/R, 9%, 8) + G (P/G 9%, 8)

$$= A\left[\frac{(1+i)^n - 1}{i\,(1+i)^n}\right] + \frac{G}{i^2}\left[\frac{(1+i)^N - Ni - 1}{(1+i)^N}\right]$$

$$= 160000\left[\frac{(1+0.9)^8 - 1}{0.09 \times (1.09)^8}\right] + \frac{(-10000)}{0.092}\left[\frac{1.09^8 - 8 \times 0.09 - 1}{(1.09)^8}\right]$$

$$= 885571.05 + (-168876.54) = 716694.51$$

PW (I) = 400000

$$\text{PW of salvage PW (S)} = 100000\,(P/F, 9\%, 8) = \frac{100000}{1.09^8} = 50186.62$$

PW of annual expenses, linear gradient G = 5000, A = 40,000

$$\text{I.e. PW (O \& M)} = A\left[\frac{(1+i)^n - 1}{i\,(1+i)^n}\right] + \frac{G}{i^2}\left[\frac{(1+i)^N - Ni - 1}{(1+i)^N}\right]$$

$$= 40000\left[\frac{(1+0.9)^8 - 1}{0.09 \times (1.09)^8}\right] + \frac{(5000)}{0.09^2}\left[\frac{1.09^8 - 8 \times 0.09 - 1}{(1.09)^8}\right]$$

$$= 221392.76 + 84438.27 = 305831.03$$

Using modified BCR $= \frac{PW(B) - PW(O \& M)}{PW(I) - PW(S)}$

$= \frac{716694.51 - 305831.03}{400000 - 50186.62}$

$= \frac{410863.48}{349813.38}$

∴ BCR = 1.174 > 1, economically feasible.

(iii) Discounted payback method

EOY	Net cash flow	PW at 9%	Cumulative PW
0	-400000	-400000	-400000
1	120000	$11009.17 = \frac{P}{(1+i)^1}$	-388990.83
2	105000	$88376.40 = \frac{P}{(1+i)^2}$	- 300614.43
3	90000	69496.51	-231117.92
4	75000	53131.89	-177986.03
5	60000	38995.88	-138990.15
6	45000	26832.03	-112158.12
7	30000	16411.027	-95747.09
8	115000	57714.62	-38032.47

Here, Payback period is more than 8 years.

19. What do you mean by financial and economic analysis? Briefly explain life cycle costing. **[2075 Baisakh]**

Solution:

Refer chapter 3.7

Life cycle costing: Life cycle costing is a system that tracks and accumulates the actual costs and revenues attributable to cost object from its invention to its abandonment. Life cycle costing involves tracing cost and revenues on a product by product base over several calendar periods.

The Life Cycle Cost (LCC) of an asset is defined as:

"The total cost throughout its life including planning, design, acquisition and support costs and any other costs directly attributable to owning or using the asset".

Life Cycle Cost (LCC) of an item represents the total cost of its ownership, and includes all the cots that will be incurred during the life of the item to acquire it, operate it, support it and finally dispose it. Life

Cycle Costing adds all the costs over their life period and enables an evaluation on a common basis for the specified period (usually discounted costs are used).

This enables decisions on acquisition, maintenance, refurbishment or disposal to be made in the light of full cost implications. In essence, Life Cycle Costing is a means of estimating all the costs involved in procuring, operating, maintaining and ultimately disposing a product throughout its life.

Thus, product life cycle costing is an approach used to provide a long-term picture of product line profitability, feedback on the effectiveness of the life cycle planning and cost data to clarify the economic impact on alternative chosen in the design, engineering phase etc.

It is also considered as a way to enhance the control of manufacturing costs. It is important to track and measure costs during each stage of a product's life cycle.

20. What are the relative methodologies of economic analysis? Explain in brief any two of them with suitable examples. **[2076 Bhadra]**

Solution: *Refer introduction part of chapter and 3.3*

21. If you planned to invest in a project which has stated following information regarding investment plan in proposal: first Cost = Rs 2 Lakhs, Project's Life = 4 years, salvage Value = Rs 50 thousands, gross Revenue = Rs 1 Lakh, O&M = Rs 30 thousands. Draw your decision based on(i) discounted payback period method (ii) equivalent worth method (iii) modified B/C ratio method and (iv) suitable rate of return method. You are provided with 14% MARR, 3 yrs. loan tenure from bank. **[2076 Bhadra]**

Solution:

First cost = Rs 2 lakhs
N = 4 years
Salvage value = Rs. 50000
Gross revenue = 1 lakh
O & M = Rs, 30000
MARR = 14% = 0.14

(i) Discounted pay back period method.

Year	Cash flow	Pw	Cumulative cash flow
0	- 200000	-200000	- 200000
1	70000	$70000(1 \text{ to } 0.14)^1 = 61403.5$	- 138596.50
2	70000	$70000\,(1+0.14)^2 = 53862.73$	- 84733.77
3	70000	$700000(1+0.14)^3 = 47248$	- 37485.76
4	70000	$700000(1+0.14)^4 = 41445.6$	3959.86

Now, by interpolation

year	cumulative cash flow
3	- 37485.76
4	3959.86
? (x)	0

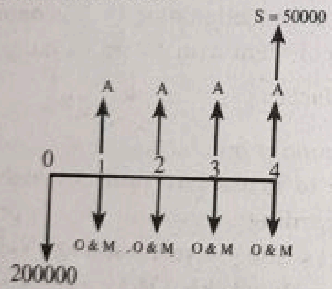
using calculator

x = 3.9 year

Therefore, payback period is 3.9 years.

(ii) Equivalent worth method

using present worth method



$$\text{Net present worth(NPW)} = -200000 + \frac{A}{(1+i)^n}\left[\frac{(1+i)^n - 1}{i}\right] - \frac{O\,\&\,M}{(1+i)^n}\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= -200000 + \frac{(100000 - 30000)}{(1+0.14)^4}\left[\frac{(1+0.14)^4 - 1}{0.14}\right] + \frac{50000}{(1+0.14)^4}$$

$$= -200000 + 2039959.86 + 29604 = 33563.87 > 0$$

Therefore B/c Project is acceptable.

(iii) Modified B/c ratio method.

$$CR = \frac{pw(B) - pw\,(0\,\&\,m)}{Pw\,(I) - pw(S)}$$

$$\text{Here, } pw\,(B) = \frac{A}{(1+i)^n}\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= \frac{10000}{(1+0.14)^4}\left[\frac{(1+0.14)^4 - 1}{0.14}\right]$$

$$= 100000 \times 2.91$$

$$= 291371.23$$

pw (0 & m) = 30000 × 2.91 = 87300

pw (s) = 29604

pw (I) = 200000

$$\therefore BCR = \frac{291371.23 - 87300}{200000 - 29604}$$

$$= 1.197 > 1 \quad \text{There fore ok.}$$

Extra Questions solutions

1. **Evaluate machine XYZ on the basis of the PW and FW methods, when the MARR is 12% per year. Pertinent cost data are as shown follows:**

Investment cost (Rs)	**150000**
Salvage value (Rs.)	**35000**
Useful life (yrs)	**15**
Annual expenses(Rs.)	**12000**
Overhead cost (Rs.) – end of 5th year	**24000**
Overhead cost (Rs.) – end of 10th year	**60000**

Solution: PW (method)

$$PW\,(12\%) = -150000 - 12000\,(P/A, 12\%, 15) - 24000\,(P/F, 12\%, 5) - 60000\,(P/F, 12\%, 10) + 35000\,(P/F, 12\%, 15)$$

$$= -150000 - 12000 \times \frac{(1.12^{15} - 1)}{1.12^{15} \times 0.12} - \frac{24000}{1.12^5} - \frac{60000}{1.12^{10}} + \frac{35000}{1.12^{15}}$$

$$= -158272.64$$

Since PW (12%) < 0 the project is not acceptable

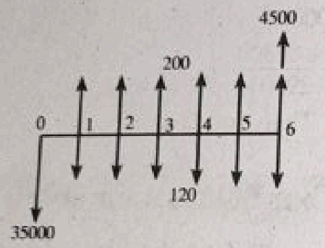
FW (method)

$$FW\,(12\%) = -150000\,(F/P, 12\%, 15) - 12000\,(F/A, 12\%, 15) - 24000\,(F/P, 12\%, 10) - 60000\,(F/P, 12\%\,5) + 35000 = -866315.72$$

Since FW (12%) < 0, the project is not acceptable

2. **Suppose that you purchased a building five years ago for Rs. 350000. Its annual maintenance expenses has been Rs. 120000 per year. At the end of five years, you sell the building for Rs. 450000. During the period of ownership you rented the building for Rs. 200000 per year at the beginning of each year. Use the AW method to evaluate this investment when your MARR is 12% per year.**

Solution:



Calculation in Thousands

AW (12%) = –3500 (A/P,12%,5) + (200 – 120) + 4500 (A/F,12%, 5)

= – Rs. 183

Since AW (12%) < 0, the project is not acceptable.

3. **If a machine will be operated according to varying hours. 1500 hrs in the first year, 2500 hrs in the second year, and 2000 hrs. In the third year. Compute the annual equivalent saving or cost per machine hour if the firm's MARR is 15% and AW(15%) = Rs. 6896**

Solution:

Let 'X' Rs/hr. be the equivalent annual saving per machine hour, which is to be determined. Equivalent annual saving in terms of X can be computed as,

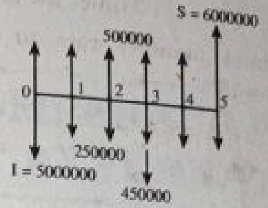
AW (15%) = [15000x (P/F, 15%, 1) + 2500x (P/F, 15%, 2) + 2000x (P/F, 15%, 3)] (A/P, 15%, 3)

Calculating, $6896 = 1975.227\,x$

$X = \text{Rs. } 3.49/\text{hr}$

4. **You purchased a building 5 years ago for Rs. 50,00,000. Annual Maintenance cost is Rs. 250,000/year. At the end of 3 years Rs. 450,000 was spent on roof repairs. At the end of 5 years you sell a building for Rs. 6000000. During the period of ownership, you rented the building for Rs. 500000 per year paid at the beginning of each year. Use AW method. MARR = 12%**

Solution:

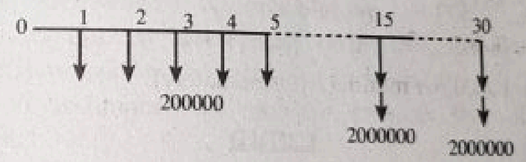


AW (12%) = –5000000 (A/P,12%, 5) – 450000 (F/P, 12% , 2) (A/F, 12%, 5) + 6000000 (A/F, 12 % 5) + 500000 – 250000 = –221520.9

AW is –ve, investment is not economically justified.

5. **Maintenance cost for a new communication tower with an expected 50 years life are estimated to be Rs. 200000 each years for the first 5 years, followed by a Rs. 2000000 expenditure in the 15th year and another Rs. 2000000 expenditure in the 30th year. If MARR = 10% what is the equivalent uniform annual cost over the entire so period?**

Solution:



Discounting all cash flows to the present

PW (10%) = 200000 (P/A, 10%, 5) + 2000000 (P/F, 10%, 15) + 2000000 (P/F, 10%, 30)

$= 200000 \times 3.7908 + 2000000 \times 0.2394 + 2000000 \times 0.0573$

$= 1351560$

AW (10%) = 1351560 (A/P, 10%,50) = Rs. 136318.34

6. **A housing company is considering building apartments at Kalanki, Kathmandu. The company has reached to a following**

No. of apartment	50
Occupancy capacity	80%
Land cost	Rs. 30000000
Construction cost	Rs. 95000000
Study period	20 years

Annual taxes & insurance 15% of total investment

Salvage values 1.2 times the land cost

Determine minimum monthly rate to be charged per unit apartment. MARR = 15%

Solution:

(Amounts are in 00000s)

AW of costs

Capital recovery costs = (300 + 950) (A/P, 15%, 20) + 300 ×1.2 × (A/F,15%,20) = Rs. 196.222

Annual taxes = 15% of (300 + 350) = Rs. 187.5

Therefore, equivalent AW costs = Rs. 383.722 (196.222 + 187.5)

AW of revenues

Let x be the monthly rent to be charged

AW of revenues = x × 12 × 50 × 80% = 480 x

Equating,

$480x = 383.722$

$x = 0.799 \approx 0.8$

x = 80000 per month (Approximately)

4 COMPARATIVE ANALYSIS OF ALTERNATIVES

- Most of engineering projects can be accomplished by more than one feasible design alternatives. The investments being required for various alternatives, may be different, and their annual revenues and cost may vary. Because different levels of investment normally produce varying economic outcomes, we must perform an analysis to determine which one of the mutually exclusive alternatives is preferred.
- In the real world of engineering practice, however, it is typical for us to have two or more choices of projects that are not independent of one another in seeking to accomplish a business objective. (As we shall see, even when it appears that we have only one project to consider, the implicit "do-nothing" alternative must be factored into the decision-making process.) In this section, we extend our evaluation techniques to multiple projects that are mutually exclusive.
- Generally, the alternatives that require the minimum investment of capital & produces satisfactory functional results will be chosen unless the incremental capital associated with an alternative having a larger investment can be justified with respect to its incremental benefits.

4.1 Comparing Mutually Exclusive Alternatives having Same Useful Life by

4.1.1 Payback period method and equivalent worth method:

- **Payback period:** In payback period, the length of time required to recover the cost of an investment for two or more alternative projects are compared and one with least payback period is selected.

(Note: Types of payback period is already studied in chapter 3)

Example: Autonumeries Company has two mutually exclusive project. Select the best project using payback period method. Study period = 5 yrs and MARR = 15%

Alternatives	A	B
Initial Investment	850000	400000
Net Annual Income	120000	110000

Take salvage value = 20% of initial investment

Solution:

For alternative A

Periods	Cash flows	PW of net cash flows (i = 15%)	Cumulative cash flows
0	- 850000	- 850000	- 850000
1	120000	104347.82	- 745652.17
2	120000	90737.24	- 654914.93
3	120000	78901.94	- 576012.98
4	120000	68610.38	- 507402.59
5	120000 + 170000	144181.25	- 36322134

[At year 5 salvage value = 170000]

Here, the cumulative Balance does changes into positive in 5 years therefore payback period is more than 5 yrs for alternative A.

For Alternative 'B'

No. of period	Cash flow	PW of net cash flow	Cumulative cash flow
0	- 400000	- 400000	- 400000
1	110000	95652.17	- 304347.82
2	110000	83175.8	- 220872.03
3	110000	72326.78	- 148545.25
4	110000	62892.85	- 85652.4
5	110000 + 80000	94463.58	8811.18

[At 5 year salvage value = 80000]

Here, cumulative balance turns positive in the year 5. So payback period lies between year 4 and year 5 by interpolation.

Payback period = 4.90 yrs.

[Note: Interpolation can be done by using calculator as in chapter 3]

Since, payback period or alternative 'B' is less than payback period of alternative 'A' the best project for company is alternative 'B'.

Equivalent worth method:

In equivalent worth method two or more alternatives are compared on the basis of (PW, AW or FW) i.e. equivalent worth. And the alternative with highest equivalent worth is selected as best alternatives.

Example: Three mutually exclusive investment alternatives for implementing an office automation plan in an engineering design form are being considered. Each alternative meets the same service requirement but capital investment & benefits are different for each alternative. The study period is 10 yrs. MAPR = 10% per year, which alternatives should be selected in view of the following investment.

Alternatives	A	B	C
Capital investment	- Rs 390,000	- Rs 920,000	- Rs 666,000
Annual cost saving	69,000	167,000	133,500

Solution:

By PW method:

PW (10%) $_A$ = - Rs 390,000 + Rs 69000 (P/A, 10%, 10)

$$= -390000 + 69000 \times \left[\frac{(1+0.1)^{10}-1}{0.1}\right] \times \frac{1}{1.1^{10}}$$

$$\left[\because F = A\left[\frac{(1+i)^n - 1}{i}\right] \text{ and } p = \frac{F}{(1+i)^n}\right]$$

$$= -390000 + 423975.13 = \text{Rs. } 33975.13$$

PW (10%) $_B$ = - Rs. 920,000 + Rs. 167,000 (P/A, 10%, 10)

$$= -920{,}000 + 167{,}000 \times \left[\frac{(1+0.1)^{10}-1}{0.1}\right] \times \frac{1}{1.1^{10}}$$

$$= \text{Rs. } 106{,}148$$

PW (10%) C = - Rs. 660,000+Rs. 133,500 (P/A, 10%, 10) = Rs. 160,304

So, Based on PW method, Alternatives C would be selected because it has largest PW value (Rs. 160,304)

By AW method

$$AW(10\%)_A = -Rs\ 390{,}000\ (A/P, 10\%, 10) + Rs\ 69{,}000$$

$$= -390{,}000 \times \left[\frac{(1+0.1)^{10} \times 0.1}{(1+0.1)^{10}-1}\right] + Rs.\ 69{,}000$$

$$\left[\because F = P(1+i)^n \ \&\ A = \frac{F \times i}{(1+i)^n - 1}\right]$$

$$= -63470.7 + 69000 = Rs.\ 5529.30$$

$$AW(10\%)_B = -920{,}000\ (A/P, 10\%, 10) + 167{,}000 = Rs.\ 17{,}316$$

$$AW(10\%)_C = -660{,}000\ (A/P, 10\%, 10) + 133{,}500 = Rs.\ 26{,}118$$

Alternative C is choose because it has largest AW value (Rs. 26,118)

By FW method:

$$FW(10\%)_A = -390{,}000\ (F/P, 10\%, 10) + 69000\ (F/A, 10\%, 10)$$

$$= -390{,}000 \times (1.1)^{10} + 69000 \times \left[\frac{(1+0.1)^{10}-1}{0.1}\right]$$

$$= Rs.\ 88{,}138$$

$$FW(10\%)_B = -920{,}000\ (F/P, 10\%, 10) + 167{,}000\ (F/A, 10\%, 10)$$

$$= Rs.\ 275{,}342$$

$$FW(10\%)_C = -660{,}000\ (F/P, 10\%, 10) + 133{,}500\ (F/A, 10\%, 10)$$

$$= Rs.\ 415{,}801$$

So, based on the FW method, the choice is again alternative C, because it has the largest FW value (Rs. 415,801).

Example: A company is planning to install a new automated plastic molding press, four different presses are available. The initial capital investment, annual expenses & salvage values are given as & (MARR = 10%). Select best one.

Press	P1	P2	P3	P4
Capital investment	- Rs. 24000	- Rs. 30,400	- Rs. 49,600	- Rs. 52,000
Useful life	5 yrs	5 yrs	5 yrs	5 yrs
Annual expenses	- Rs. 31,200	- Rs. 29,128	- Rs. 25,192	- Rs. 22,880
Salvage	10,000	18000	20000	25000

Solution:

Here we solve (By the FW method):

$$FW(10\%)_{P1} = -24000\ (F/P, 10\%, 5) - 31200\ (F/A, 10\%\ 5) + 10000$$

$$= -24000 \times (1+0.1)^5 - 31200\left[\frac{(1+0.1)^5-1}{0.1}\right] + 10000$$

$$= -229{,}131 + 10000$$

$$= -219131$$

Similarly,

$$FW(10\%)_{P2} = -30{,}400\ (F/P, 10\%, 5) - 29.128\ (F/A, 10\%, 5) + 18000$$

$$= -226{,}788 + 18000$$

$$= -208788$$

$$FW(10\%)_{P3} = -49{,}600\ (F/P, 10\%, 5) - 25{,}192\ (F/A, 10\%, 5) + 20000$$

$$= -233{,}689 + 20000$$

$$= -213689$$

$$FW(10\%)_{P4} = -52{,}000\ (F/P, 10\%, 5) - 22{,}880\ (F/A, 10\%, 5) + 25000$$

$$= -223{,}431 + 25000$$

$$= -198431$$

So, Based on FW analysis preference ranking is (P4 > P2 > P3 > P1) and the best option is press 4.

[Note: It can solve by using PW and FW method also]

4.1.2 Rate of Return method and Benefit cost ratio method:

The projects having highest value of IRR, ERR and BCR may not be the preferred alternative, they are just relative measure and cannot be in the way as (PW, AW and FW) method. For this purpose we adopt incremental analysis. For independent projects, no incremental analysis is necessary.

Procedure for incremental analysis:

1. Arrange the feasible alternative, based on increasing capital investment.
2. Establish a base alternative:

a. Cost alternative: The first alternative (Least capital investment) is base.

b. Investment alternatives: If the first alternative is acceptable (IRR > MARR), select it as the base. If the first alternative is not acceptable, choose the next alternative in order of increasing capital investment & check the profitability criteria (PW etc) values. Continue until an acceptable alternative is obtained. If none is obtained, the do nothing alternative is selected.

3. Use iteration to evaluate differences between alternative until no more alternative exist.

a) If the incremental cash flow between the next (higher capital investment) alternative and the current selected alternative is acceptable, choose the next alternative as the current best alternative. Otherwise, retain the last acceptable alternative as the current best.

b) Repeat, and select the preferred alternative the last one for which the incremental cash flow was acceptable.

Example: The estimated capital investment and the annual expenses for four alternatives design of a diesel powered air compressor are shown. The study period is five years and the MARR is 20% per year. Based on this information determine the preferred design alternative using IRR on incremental investment.

Design alternatives	D1	D2	D3	D4
Capital investment	- Rs. 100000	- Rs. 140600	- Rs. 148200	- Rs. 122000
Annual expenses	- 29000	- 16900	-14800	- 22,100
Useful Life	5	5	5	5
Market value	10,000	14,000	25,600	14,000

[Note: Market value is salvage value]

Solution:

Arrange the four mutually exclusive cost alternatives based on their increasing capital investment costs.

So order of alternatives for incremental analysis is D1, D4, D2 and D3.

Take D1 as the base alternatives

Now, the first increment analysis is between D1 and D4 i.e. D4 - D2 and present in tabular form.

Increment covered	D4 - D1	D2 - D4	D3 - D4
Capital investment	- 22000	- 18.600	- 26.200
Annual expenses	6900	5200	7,300
Market value	4000	0	11,600

Calculate IRR of increment cash flow of D4 - D1

$$PW(i\%) = 0$$

$$-22000 + 6900\,(P/A, i\%, 5) + 4000\,(F/P, i\%, 5) = 0$$

$$-22000 + 6900\left[\frac{(1+i)^5 - 1}{i}\right] \times \frac{1}{(1+i)^5} + 4000 \times \frac{1}{(1+i)^5} = 0$$

Solving by calculator,

$i = 20.5\% > \text{MARR } (20\%)$ So, eliminate D1

Again, incremental analysis is between D4 and D2.

Calculate IRR of incremental cash flow of D2 - D4.

$FW(i\,\%) = 0$ [∵ for calculating IRR any of AW, PW & FW is made equal to zero]

$$-18{,}600\,(F/P, i\%, 5) + 5200\,(F/A, i\%, 5) + 0 = 0$$

$$-18{,}600 \times (1+i)^5 + 5200\left[\frac{(1+i)^5 - 1}{i}\right] = 0$$

Solving by calculator

$i = 12.3\% <$ MARR (20%0 So eliminate D2.

Again, make incremental analysis between D3 and D4.

Calculate IRR of incremental cash flow of D3 - D4.

$$FW(i\,\%) = 0$$

$$-26{,}200\,(F/P, i\%, 5) + 7{,}300\,(F/A, i\%, 5) + 11{,}600 = 0$$

$$-26{,}200 \times (1+i)^5 + 7300\left[\frac{(1+i)^5 - 1}{i}\right] + 11600 = 0$$

Solving by calculator

$i = 20.4\% > \text{MARR } (20\%)$ So eliminate D4 and Select D3 as the best alternative.

Example: An engineering firm is considering following exclusive alternatives.

EOY	Alternatives		
	A1	A2	A3
0	- 4000	- 2000	- 6000
1	3000	1600	3000
2	2000	1000	4000
3	1600	1000	2000

Determine the best alternatives by using IRR incremental analysis.

Assuming MARR = 19%

Solution: This is the case of investment alternatives so,

First calculate IRR for all alternatives

For A1 PW (i %) = 0

$$-4000 + \frac{3000}{(1+i)} + \frac{2000}{(1+i)^2} + \frac{1600}{(1+i)^3} = 0$$

∴ I = 34.34% > MARR, (accepted)

Similarly for A2 PW (i %) = 0

$$-2000 + \frac{1600}{(1+i)} + \frac{1000}{(1+i)^2} + \frac{1000}{(1+i)^3} = 0$$

i = 40.76% > MARR, (accepted)

& for A3 PW (i %) = 0

∴ i = 24.81% > MARR (accepted)

Taking A2 as the base alternative & order for incremental analysis is A2, A1, and A3.

Now, the first increment analysis is between A1 and A2 i.e. A1-A2

EOY	A1 - A2	A3 - A1
0	- 2000	- 2000
1	1400	0
2	1000	2000
3	600	400
IRR	27.61%	8.8%

Calculate IRR of incremental cash flow of (A1 - A2)

PW (i %) = 0

$$-2000 + \frac{1400}{(1+i)} + \frac{1000}{(1+i)^2} + \frac{600}{(1+i)^3} = 0$$

Solving by calculator

i = 27.61% > MARR (15%); accept A1 & eliminate A2

Again, increment analysis between A1 & A3 i.e. (A3 - A1)

PW (i %) = 0

$$-2000 + 0 + \frac{2000}{(1+i)^2} + \frac{400}{(1+i)^3} = 0$$

i = 8.8% < MARR (15%); accept A1, Eliminate A3.

So the best alternative is A1.

4.2 Comparing Mutually Exclusive Alternatives having Different Useful Lives

When the useful lives of mutually exclusive alternatives are different, the repeatability assumption may be used in their comparison if the study period can be infinite in length or a common multiple of the useful lives.

4.2.1 Repeatability Assumption

Two or more alternatives having different useful life are changed into projects having same useful life by expanding their life up to at least common year.

The economic consequences that are estimated to happen in an alternative's initial useful life span will also happen in all succeeding life spans replacements)

Actual situations in engineering practice seldom meet both conditions.

Example: The following data have been estimated for two mutually exclusive investment alternatives A and B, associated with a small engineering project for which revenues as well as expenses are involved. They have useful lives of four and six years respectively. If the MARR = 10% per year, show which alternative is more desirable by

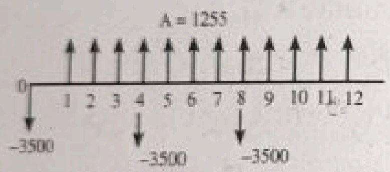
using equivalent worth method. Use repeatability assumptions.

Alternative	A	B
Capital investment	- Rs. 3500	- Rs. 5000
Annual revenue	1900	2500
Annual expenses	- 645	- 1020
Useful life (year)	4	6
Market value at the end of useful life	0	0

Solution: The LCM of useful lives of alternatives A and B is 12 years. Using the repeatability assumptions and a 12 years study period.

For Alternative A

Net revenue = Revenue - expenses = 1900 - 645 = Rs.1255



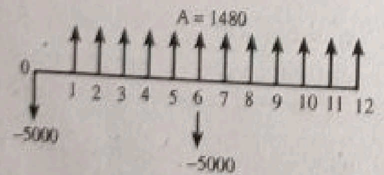
Now, PW (10%) of A = - 3500 - 3500 [(P/F, 10%,4) + (P/F,10%,8)] + 1255 (P/A,10%,12)

$$= -3500 - 3500\,[(1+0.1)^{-4} + (1+0.1)^{-8}] + 1255 \times \left[\frac{1.1^{12}-1}{0.1}\right] \times \frac{1}{1.1^{12}}$$

= Rs. 1028

For alternative B,

Net revenue = Revenue - expenses = 2500 - 1020 = Rs. 1480



Now, PW (10%) of B = - 5000 - 5000 (P/F, 10%,6) + 1480 (P/A, 10%,12)

= Rs. 2,262

Based on PW method we would select alternative B because it has larger value (2,262)

[Note: Similarly we can apply AW & FW method also]

4.2.2 Co-terminated Assumption:

- A finite and identical study period is used for all alternatives.
- This planning horizon, combined with appropriate adjustments to the estimated cash flows, puts the alternatives on a common and comparable basis.
- Used when repeatability assumption is not applicable.
- This is the approach most frequently used in engineering practice.

a) Useful life shorter than study period (useful life < study period)

When project lives are shorter than the required service period, we must consider how, at the end of the project lives, we will satisfy the rest of the required service period. Replacement projects—additional projects to be implemented when the initial project has reached the limits of its useful life—are needed in such a case. A sufficient number of replacement projects that match or exceed the required service period must be analyzed.

To simplify our analysis, we could assume that the replacement project will be exactly the same as the initial project, with the same costs and benefits. However, this assumption is not necessary. For example, depending on our forecasting skills, we may decide that a different kind of technology—in the form of equipment, materials, or processes—is a preferable replacement. Whether we select exactly the same alternative or a new technology as the replacement project, we are ultimately likely to have some unused portion of the equipment to consider as salvage value, just as in the case when the project lives are longer than the analysis period. Of course, we may instead decide to lease the necessary equipment or subcontract the remaining work for the duration of the analysis period. In this case, we can probably match our analysis period and not worry about salvage values.

In any event, at the outset of the analysis period, we must make some initial guess concerning the method of completing the analysis. Later, when the initial project life is closer to its expiration, we may revise our analysis with a different replacement project. This is only reasonable, since economic analysis is an ongoing activity in the life of a company and an investment project, and we should always use the most reliable, up-to date data we can reasonably acquire.

i) **Cost alternatives:** Each cost alternative must provide same level of service as study period:

1) Control for service or least equipment for remaining time.
2) Repeat part of useful life of original alternative until study period ends.

ii) **Investment alternatives:** Assume all cash flows reinvested in other opportunities of MARR to end of study period.

This is mostly applicable in our syllabus numerical problems.

Example: The Smith Novelty Company, a mail-order firm, wants to install an automatic mailing system to handle product announcements and invoices. The firm has a choice between two different types of machines. The two machines are designed differently, but have identical capacities and do exactly the same job. The $12,500 semiautomatic model A will last three years, while the fully automatic model B will cost $15,000 and last four years. The expected cash flows for the two machines, including maintenance, salvage value, and tax effects, are as follows

n	A	B
0	-$12500	-$15000
1	-5000	-4000
2	-5000	-4000
3	-5000 + 2000	-4000
4		-4000 + 1500
5		

As business grows to a certain level, neither of the models may be able to handle the expanded volume at the end of year 5. If that happens, a fully computerized mail-order system will need to be installed to handle the increased business volume. In the scenario just presented, which model should the firm select at MARR=15%.

Solution: Given: Cash flows for two alternatives as shown in Figure below, analysis period of five years, and i=15%.

Find: NPW of each alternative and which alternative to select.

Since both models have a shorter life than the required service period of 5 years, we need to make an explicit assumption of how the service

requirement is to be met. Suppose that the company considers leasing equipment comparable to model A at an annual payment of $6,000 (after taxes) and with an annual operating cost of $5,000

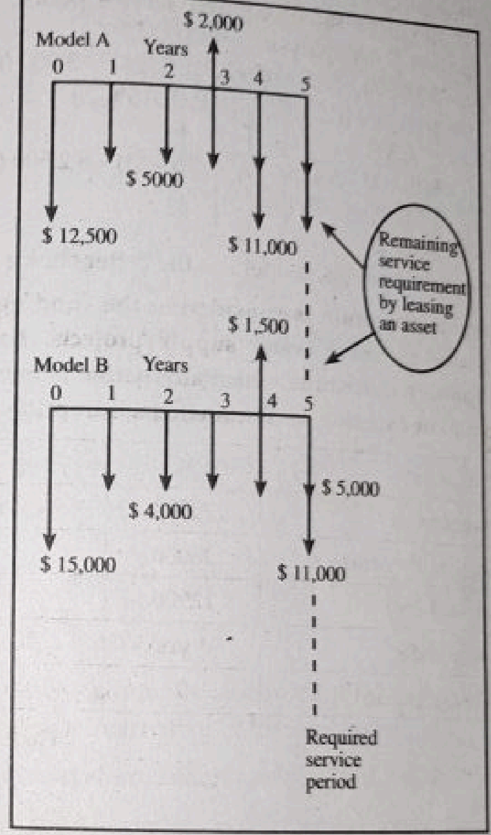


Figure: Comparison for service projects with unequal lives when the required service period is longer than the individual project live

For the remaining required service period. In this case, the cash flow would look like

n	Model A	Model B
0	-$ 12,500	-$15,000
1	-5,000	-4,000
2	-5,000	-4,000
3	-5,000 + 2,000	-4,000
4	-5,000– 6,000	-4,000 + 1,500
5	-5,000 – 6,000	-5,000 + 6,000

Here, the boxed figures represent the annual lease payments. (It cost $ 6,000 to lease the equipment and $ 5,000 to operate it annually. Other maintenance costs will be paid by the leasing company.) Note that both alternatives now have the same required service period of five years. Therefore, we can use NPW analysis.

$$PW(15\%)_A = -\$12{,}500 - \$5{,}000\,(P/A, 15\%, 2) - \$3{,}000\,(P/F, 15\%, 3) - \$11{,}000\,(P/A, 5\%, 2)\,(P/F, 15\%, 3)$$

$$= -\$34{,}359$$

$$PW(15\%)_B = -\$15{,}000 - \$4{,}000\,(P/A, 15\%, 3) - \$2{,}500\,(P/F, 15\%, 4) - \$11{,}000\,(P/F, 15\%, 5)$$

$$= -\$31{,}031$$

Since these are service projects, model B is the better choice

Example: A large corporation is considering the funding of following mutually exclusive water supply projects. Based on given information determine which alternative is more desirable to implement by using co-terminated assumption.

Alternative	A	B
Investment	70000	100000
Annual Revenue	38000	50000
Annual cost	12900	27660
Useful life	4 yrs	8 yrs
Salvage value	0	0

[Take MARR = 10%]

Solution:

Taking study period as 8 years

(Note: always take study period equal or greater than useful lives of all alternatives, if not given in the question]

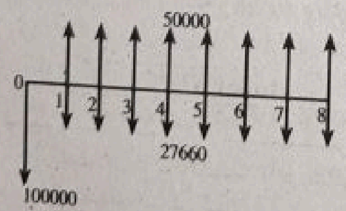


Fig.: Cash Flow Diagram of Project B

No adjustment is required for alternative B because study period is equal to useful life.

Now, $FW(10\%)B = -10000[F/P, 10\%, 8] + (50000 - 27660)\,[F/A, 10\%, 8]$

$$= -10000\,(1+0.1)^8 + 22340\left[\frac{(1+0.1)^8 - 1}{0.1}\right]$$

$$= \text{Rs. } 41118.85$$

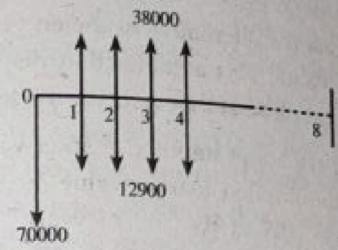


Fig: Cash Flow Diagram of Project A

For alternative A, the cash flow accumulated at end of useful life is reinvested for extended periods.

$$FW(10\%) = [-70000\,(F/P, 10\%, 4) + (38000 - 12900)\,(F/A, 10\%\ 4)] \times [F/p, 10\%\ 4]$$

$$= [-70000\,(1+0.1)^4 + 25100 \times \left(\frac{1.1^4 - 1}{0.1}\right)] \times (1+0.1)^4$$

$$= \text{Rs. } 20500.47$$

Here based on co-terminated assumption FW for project B is higher (Rs 41118.85). So select alternative 'B' for implementation.

b) Study period shorter than useful life: (Useful life > study period)

Project lives rarely conveniently coincide with a firm's predetermined required analysis period; they are often too long or too short. The case of project lives that are too long is the easier one to address.

Consider the case of a firm that undertakes a five-year production project when all of the alternative equipment choices have useful lives of seven years. In such a case, we analyze each project for only as long as the required service period (in this case, five years). We are then left with some unused portion of the equipment (in this case, two years' worth), which we include as salvage value in our analysis. **Salvage value** is the amount of money for which the equipment could be sold after its service to the project has been rendered. Alternatively, salvage value is the dollar measure of the remaining usefulness of the equipment.

A common instance of project lives that are longer than the analysis period occurs in the construction industry: A building project may have a relatively short completion time, but the equipment that is purchased has a much longer useful life.

When the study period is shorter than the useful life of any project or an asset, we truncate the alternative at the end of study period using an estimated market value. This assumes that disposable assets will be sold at the end of study period at that value.

For this purpose we adopt imputed market value technique, which is sometimes called the implied market value & estimated by imputed market value (IMV_T) = [PW at the end of year T of remaining capital recover amounts] + [PW at the end of T of original market value at the end of useful life]

Imported market value Technique: Used for useful life > study period

When current market place data is unavailable for an asset, it is sometimes necessary to estimate the market value of an asset. Also termed as implied market value.

Estimating is based on logical assumption about the remaining life for the asset.

IMV_T = [EW at the end of year T of remaining capital recovery amounts] + [EW at the end of year T of original market value at the end of useful life]

$IMV = EW_{CR} + EW_{MV}$

Where,

IMV = Imputed market value of study period.

EW_{CR} = Equivalent worth of capital recovery

EW_{MV} = Equivalent worth of market value.

Example: Wastage Management Company (WMC) has own a contract that requires the firm to remove radioactive material from govern - owned property and transport it to a designated dumping site. This task requires a specially made ripper-bulldozer to dig and load the material onto a transportation vehicle. Approximately 400,000 tons of waste must be moved in a period of two year.

- **Model A costs \$ 150,000 and has a life of 6,000 hours before it will require any major overhaul. Two units of model A would be required to remove the material within two years, and the operating cost for each unit would run to \$ 40,000/year for 2,000 hours of operation. At this operational rate, the model would be operable for three years, at the end of which time it is estimated that the salvage value will be \$ 25,000 for each machine.**
- **A more efficient model B costs \$240,000 each, has a life of 12,000 hours without any major overhaul, and costs \$ 22,500 to operate for 2,000 hours per year to complete the job within two years. The estimated salvage value of model B would be required to remove the material within two years.**

Since the lifetime of either model exceeds the required service period of two years (Figure...), WMC has to assume some things about the used equipment at the end of that time. Therefore, the engineers at WMC estimate that, after two years, the model A units could be sold for \$ 45,000 each and the model B units for \$ 125,000 each. After considering all tax effects, WMC summarized the resulting cash flows (in thousands of dollars) for each project as follows.

Annual Equivalent Cost ($)

Period	Model A		Model B	
0	- \$300		- \$480	
1	-80		-45	
2	-80	+90	-45	+250
3	-80	+50	-45	
4			-45	
5			-45	
6			-45	+60

Here, the figures in the boxes represent the estimated salvage values at the end of the analysis period (the end of year 2). Assuming that the firm's MARR is 15%, which option would be acceptable?

Solution:

Given,

Cash flows for two alternatives as shown in Figure here and I =15% per year. Find: NPW for each alternative and which alternative is preferred.

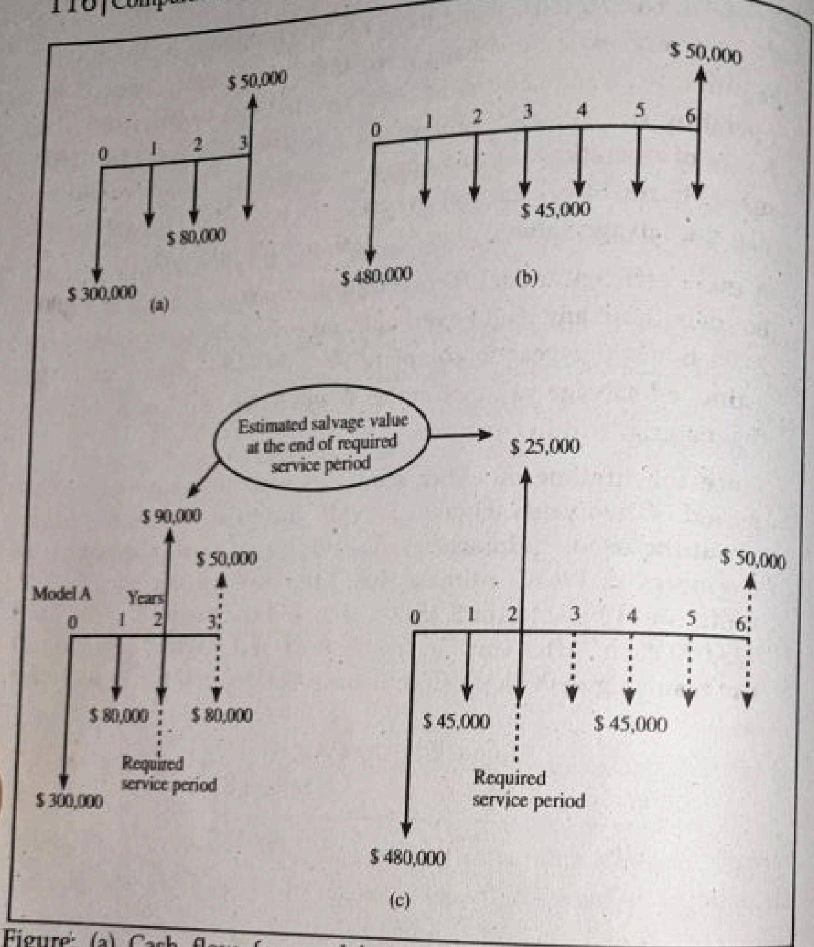


Figure: (a) Cash flow for model A: (b) cash flow for model B; (c) comparison of service projects with unequal lives when the required service period is shorter than the individual project life

First, note that these are service projects, so we can assume the same revenues for both configurations. Since the firm explicitly estimated the market values of the assets at the end of analysis period (two years), we can compare the two models directly. Because the benefits (removal of the waste) are equal, we can concentrate on the costs.

$PW(15\%)_A = -\$300 - \$80\,(P/A, 15\%, 2) + \$\,90\,(P/F, 15\%, 2)$

$= -\$362;$

$PW(15\%)_B = -\$480 - \$45(P/A, 15\%, 2) + \$250(P/F, 15\%, 2)$

$= -\$364$

Model A has the least negative PW costs and thus would be preferred

Example: Use co-terminate (i.e. the imputed market value technique) method to develop an estimated market value and determine the best alternative project by taking study period of 5 yrs for given information.

Alternative	X	Y
Capital investment	- 35000	- 50000
Annual expenses	1500	2500
Annual revenue	13000	17500
Salvage Value	3500	5000
Useful life	5 years	8 years

Use MARR = 10% per year

Solution:

For alternative X:

Net annual revenue = Revenue - expenses

= 13000 - 1500 = 11500

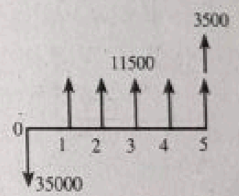


Fig. Cash Flow Diagram of X

$FW(10\%)_X = -35000\,(F/P, 10\%, 5) + 11500\,(F/A, 10\%, 5) + 3500$

$= -35000\,(1 + 0.1)^5 + 11500\left[\frac{1.1^5 - 1}{0.1}\right] + 3500 = \text{Rs. } 17340.8$

For alternative Y:

Net annual revenue = 17500 - 2500 = 15000

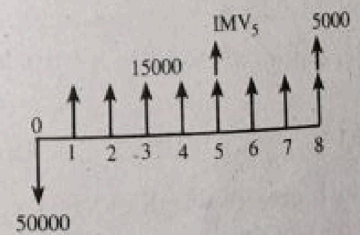


Fig. Cash Flow Diagram of Y

By using imputed market value (IMV) technique

Capital recovery, CR (10%) = 50000 (A/P, 10%,8) – 5000(A/F, 10%,8)

$$= 50000\,(1.1)^8 \times \frac{0.1}{1.1^8 - 1} - 5000 \times \frac{0.1}{1.1^8 - 1}$$

$$= 9372.2 - 437.2$$

$$= \text{Rs. } 8935$$

Now, PW at the end of 5th year for remaining 3 years,

$$PW\,(10\%)_{CR} = 8935\,(P/A, 10\%, 3) = 8935 \times \left[\frac{1.1^3 - 1}{0.1}\right] \times \frac{1}{1.1^3}$$

$$= \text{Rs. } 22220$$

Again, PW at the end of 5th year of original market value at the end of useful life (8 years)

I.e. at the end of 5th year of salvage value (Rs. 5000)

So, PW (10%) MV = 5000 (P/F, 10%, 3) = 5000 × $(1 + 0.1)^{-3}$ = Rs. 3756.50

Now, imputed Market value (IMV) = 22220 + 3756.5 = Rs. 25976.5

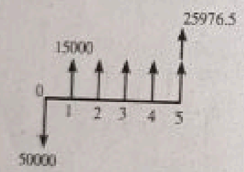


Fig: Revised Cash Flow Diagram of Y

$$FW\,(10\%)_y = -50000\,(F/P, 10\%, 5) + 15000\,(F/A, 10\%\ 5) + 25976.5$$

$$= -50000 \times (1 + 0.1)^5 + 15000\left[\frac{(1 + 0.1)^5 - 1}{0.1}\right] + 25976.5$$

$$= \text{Rs. } 37027.5$$

Based on co-terminated assumption FW for alternative Y is higher (Rs. 37027.5)

So, alternative Y is best selection.

4.2.3 Capitalized worth method:

- The process in which present worth of all revenues and for expenses over an infinite length of time is calculated is called capitalized worth (CW) method.

- The amount of money that must be invested today to yield a certain return A at the end of each and every period assuming interest rate i% is capitalized cost.
- This is a convenient basis for comparing mutually exclusive alternatives when the period of needed service is indefinitely long end repeatability assumption is applicable.
- It is present worth of project which lasts forever such as:
  -Public sector projects
  -Bridge
  -Hydroelectric dams
  -Irrigation system
- The CW of a perpetual series of end of period uniform payments A, with interest at i% per period is A (P/A, i%, ∞)

$$\text{So, } CW\,(i\%) = PW_{N\to\infty} = A(P/A, i\%, \infty) = A\left[\lim_{N\to\infty} \frac{(1+i)^N - 1}{i(1+i)^N}\right] = A\left(\frac{1}{i}\right)$$

$$\therefore \boxed{CW\,(i\%) = \frac{A}{i}}$$

Hence the CW of a project with interest i% per year is the annual equivalent of the project over its useful life divided by i.

Example: A selection is to be made between two structured designs. Because revenues do not exist, only negative cash flow amounts (cost) and the market value at the end of useful life are situated as follows:

Alternative	Structure M	Structure N
Capital investment	- 12,000	40000
Market value	0	10000
Annual expenses	- 22000	- 1000
Useful life	10 yrs	25 yrs

By using the repeatability assumption & the CW method of analysis, determine which structure is better if the MARR is 15% per year.

Solution: The annual worth (AW) over the useful life of each alternative structure at MARR = 15%, is calculated as,

$$AW\,(15\%)\,M = -12000\,(A/P, 15\%, 10) - 2200$$

$$= -12000 \times (1.15)^{10} \times \frac{0.15}{1.15^{10} - 1} - 2200 = -4592$$

AW (15%) N $= -40000(A/P, 15\%, 25) + 10000 (A/F, 15\%, 25) - 1000$

$= -40000 \times (1.15)^{25} \times \frac{0.15}{1.15^{25}-1} + 10000 \times \frac{0.15}{1.15^{25}-1} - 1000$

$= -7,141$

Then the CWs of structure M & N are as follows:

$CW(15\%)M = \frac{AW_M}{i} = \frac{-4592}{0.15} = -30,613$

$CW(15\%)N = \frac{AW_N}{i} = \frac{-7141}{0.15} = -47,607$

So, based on the CW of each structural design, alternative M should be selected because it has the lesser negative value of CW (-30,613)

4.3 Comparing Mutually Exclusive, Contingent and Independent Projects in Combination

Mainly there are three major groups of investment opportunities

a) Mutually exclusive: At most one project out of the group can be chosen.

b) Independent: The choice of a project is independent of the choice of any other project in the group, so that all or none of the projects may be selected or some number in between.

c) Contingent: The choice of a project is conditional on the choice of one or more other projects. Contingent project is one of acceptance of one require the acceptance of another

Formation of mutually exclusive alternatives:

1) If A, B are two independent projects then the mutually exclusive combination is:

Mutually exclusive combination	A	B	Remarks/Decision
1	0	0	Do nothing
2	1	0	Accept A
3	0	1	Accept B
4	1	1	Accept Both A & B

2) If A, B, C is three projects where C is contingent on acceptance of B and acceptance of B is contingent of acceptance of A, we can make following combination.

Mutually exclusive combination	A	B	C	Decision
1	0	0	0	Do nothing
2	1	0	0	Accept A
3	1	1	0	Accept A & B
4	1	1	1	Accept A, B & C

Example: The following are five proposed projects being considered by an engineer in an integrated transportation company for upgrading an intermodal shipment transfer facility for less than truckload lots of consumer goods. The inter relationships among the projects are:

Project B1
Project B2 } mutually exclusive and independent of C set

Project C1
Project C2 } mutually exclusive and dependent (contingent) on the acceptance of B2.

Project D is contingent on the acceptance of C1.

The respective cash flow is shown on table. Using PW method & MARR = 10% per year, determine what combination of projects is best if the capital to be invested is (a) unlimited and (b) limited to 48000.

Cash flows for end of year (Rs.)

Projects/EOY	0	1	2	3	4
B1	- 50000	20000	20000	20000	20000
B2	- 30000	12000	12000	12000	12000
C1	- 14000	4000	4000	4000	4000
C2	- 15000	5000	5000	5000	5000
D	- 10000	6000	6000	6000	6000

Solution:

The PW for each project is calculated as

PW (10%) B1 $= -50{,}000 + 20000\,(P/A, 10\%, 4)$

$$= -50000 + 20000 \times \left[\frac{1.1^4 - 1}{0.1}\right] \times \frac{1}{1.1^4} = 13{,}400$$

Similarly,

PW (10%) B2 = −30000 + 12000 (P/A, 10%, 4) = 8000

PW (10%) C1 = −14000 + 4000 (P/A, 10%, 4) = −1300

PW (10%) C2 = −15000 + 5000 (P/A, 10%, 4) = 800

PW (10%) D = −10000 + 6000 (P/A, 10%, 4) = 9000

Mutually exclusive project combinations is:

Mutually exclusive combination	B1	B2	C1	C2	D
1	0	0	0	0	0
2	1	0	0	0	0
3	0	1	0	0	0
4	0	1	1	0	0
5	0	1	0	1	0
6	0	1	1	0	1

Combined project cash flows and PW is:

Mutually exclusive combination	Cash flow for EOY (× 1000)					Invested capital (× 1000)	Pw (× 1000)
	0	1	2	3	4		
1	0	0	0	0	0	0	0
2 (B1)	−50	20	20	20	20	50	13.4
3 (B2)	−30	12	12	12	12	30	8.0
4 (B2, C1)	−44	16	16	16	16	44	6.7
5 (B2, C2)	−45	17	17	17	17	45	8.9
6 (B2, C1, D)	−54	22	22	22	22	54	15.7

Note: Here, for combination 1 no project is selected.

For combination 2, project B1 is selected so, combined cash flows is same as of B1.

For combination 3, project B2 is selected so, combined cash flows is same as of B2

For combination 4, project B2 & C1 is selected so, combined cash flow is (cash flow of B2 + cash flow of C1)

Similarly for combination 5 and 6

PW calculation for combined project is:

PW (10%) of 1 = 0

PW (10%) of 2 = − 50 + 20 (P/A, 10%, 4) = 13.4

PW (10%)3 = − 30 + 12 (P/A, 10%, 4) = 8.0

PW (10%)4 = − 44 + 16 (P/A, 10%, 4) = 6.7

Similarly,

PW (10%)5 = 8.9

PW (10%)6 = 15.7

(All these values shown in above table)

So,

a) Based on mutually exclusive combination the best combination is 6(B2, C1, D) which has the highest PW if capital available is unlimited.

b) If, capital available is limited to 48000 mutually exclusive combination 2 & 6 are not feasible. Of the remaining mutually exclusive combinations, 5 is best, means combination of B2 and C2 is selected with a PW = 8.9 × 1000 = 8900

Old Question Solution

1. **From the following information select the best project.**

	Project A	Project B
Initial investment	35000	50000
Annual revenue	16450	25000
Annual cost	3000	13,830
Useful life	4 yrs	8 yrs
Salvage value at the end of useful life	0	0

MARR = 10%

When service period required is

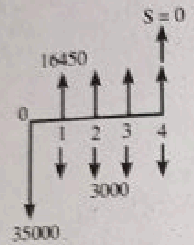
(i) 4 years by FW method

(ii) 8 years by IRR method with PW formulation [2069 Bhadra]

Solution:

i) 4 years by FW method

For Project A



$$PW(10\%) = -35000 + (16450 - 3000)\ (P/A, 10\%, 4)$$

$$= -35000 + 13450 \times \frac{(1.1^4 - 1)}{0.1} \times \frac{1}{1.1^4}$$

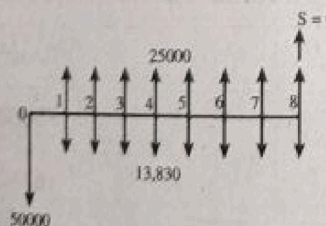
$$= -35000 + 13450 \times 3.169$$

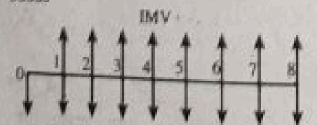
$$= \text{Rs. } 7634.69$$

$$\therefore FW = 7634.69 \times 1.1^4 = \text{Rs. } 11177.95$$

(Or we can direct calculate FW instead of PW)

__For Project B:__ Study period (4 yrs) is less than useful life so, use co-terminated assumption. (Calculating imputed market value)





IMV = (PW at end of year T of remaining capital recovery amount) + (PW at end of year T of original market value at the end of useful life)

For savage value = 0

Capital recovery (CR) = 50000 (A/P, 10%, 8)

$$= 5000 \times 1.1^8 \times \frac{0.1}{1.1^8 - 1} = \text{Rs. } 9372.2$$

PW at the end of 4th year for remaining 4 years

$$PW(10\%)_{CR} = \text{Rs. } 9372.2\ (P/A, 10\%\ 4)$$

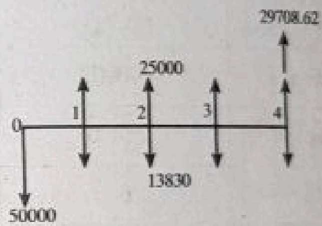
$$= 9372.2 \times \frac{1.1^4 - 1}{0.1} \times \frac{1}{1.1^4} = \text{Rs. } 29708.62$$

Salvage value is zero so,

PW at end of 4th year of original market value = 0

So, IMV = 29708.62 + 0 = Rs. 29708.62

Now,



$$\therefore PW(10\%)4 = -50000 + (25000 - 13830)(P/A, 10\%, 4) + 29708.62\ (P/F, 10\%, 4)$$

$$= -50000 + 11170 \times 3.169 + 29708.62 \times 0.6383 = \text{Rs. } 5688.29$$

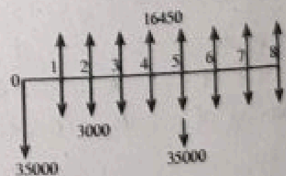
$$\therefore FW = 5688.29 \times 1.1^4 = \text{Rs. } 8328.22$$

Here $FW(10\%)_A > FW(10\%)_B$

So, select project A having higher FW.

(ii) 8 year by IRR with PW formulation

__For Project A:__



Considering IRR = i%

Then $PW(i\%)_8 = 0$

$$-35000 - \frac{35000}{(1+i)^4} + (16450 - 3000)\frac{(1+i)^8 - 1}{i} \times \frac{1}{(1+i)^8} = 0$$

By solving calculator,

$i = 19.73\%$

For Project B:

$PW(i,\%)_B = 0$

$$-50000 + (25000 - 13830)\frac{(1+i)^8 - 1}{i} \times \frac{1}{(1+i)^8} = 0$$

By using calculator, $i = 15.08\%$

So $IRR_A > IRR_B$

So select project A.

2. **State and explain about the cases of mutually exclusive contingent and independent projects with example. Compare the following projects by using repeatability assumption when MARR = 12%**

Project	A	B
Initial Investment	200000	300000
Annual revenue	25000	30000
Annual costs	7000	9000
Useful life	6 yrs	8 yrs
Salvage value	90000	20000

(2070 Magh)

Solution:

Mutually exclusive project: At most one project, out of no. of alternatives to achieve desired goal can be chosen. This kind of opportunity is called mutually exclusive.

Contingent: Among the available alternative, the choice of one project is conditional on the choice of one or more other projects is called contingent. E.g.: Purchasing of printer is dependent on purchasing of computer.

Independent: The choice of a project is independent of the choice of any other project in the group. i.e. Project can be accepted or rejected without influencing the acceptance or rejection of other.

E.g.: Buying machinery, furniture & truck are 3 independent projects.

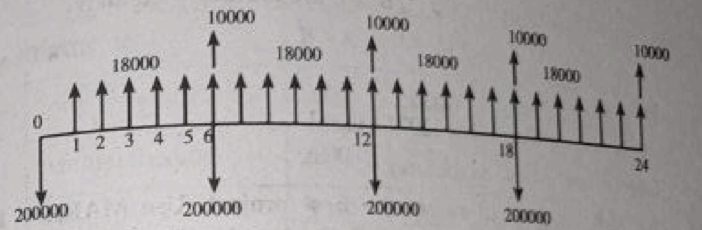
Here, common study period for project A & B is = LCM of 6 & 8

= 24 years.

Using repeatability assumption:

For project A: Net Revenue = Annual Revenue – Annual costs

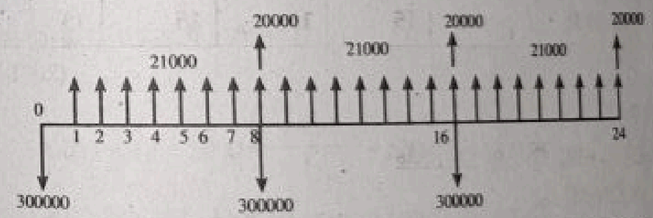
= Rs. 18000



Future worth, $FW(12\%)_{24}$ = –20000 [(F/P,12%,24) + (F/P,12%,18) + (F/P,12%,12) + (F/P,12%,6)] + 10000 [(F/P,12%,18) + (F/P,12%,12) + (F/P,12%,6) + 1] + 18000 (F/A,12%,24)

$$= -200000\,[1.12^{24} + 1.12^{18} + 1.12^{12} + 1.12^{6}] + 10000\,[1.12^{18} + 1.12^{12} + 1.12^{6} + 1] + 18000 \times \frac{1.12^{24} - 1}{0.12} = \text{Rs. } 3476118.53$$

For Project B: Net Revenue = 30000 – 9000 = 21000



PW (12%)24 = –300000 [(F/P,12%,24) + (F/P,12%,16) + (F/P,12%,8)]

+ 20000 [(F/P,12%,16) + (F/P,12%,8) + 1]

+ 21000 (F/A,12%,24)

= – 300000 [15.17 + 6.13 + 2.476] + 20000 [6.13 + 2.476 + 1]

+ 21000 × 118.15

= – Rs. 4459530

Here FW of A > FW of B

So Project A would be best selection

3. **Use IRR method to select best project MARR = 12%**

	A	B	C	D
Initial investment	1100	1500	2750	2000
Annual income	500	700	1200	950
Useful life	4	4	4	4
Salvage value	250	500	800	1000

Select the best combination if A, B & C are mutually exclusive

(2070 Bhadra)

Solution:

Solution is similar to the [2071 Magh]

Refer to next question (below):

4. **Use IRR method to select the best project. Use MARR = 12%. Select best combination if A, B & C are mutually exclusive.**

	A	B	C	D
Initial investment	10000	15000	27000	20000
Annual income	5000	7000	12000	9000
Useful life	4	4	4	4
Salvage value	2500	5000	8000	10000
MARR	15	15	15	15

(2071 Magh)

Solution:

Calculating IRR of Projects

For Project A:

FW (i%) = 0

$$-10000 \times (1+i)^4 + 5000 \times \frac{(1+i)^4 - 1}{i} + 2500 = 0$$

$\therefore$ i = 39.34% > MARR, accepted

For Project B:

FW (i%) = 0

$$-15000 \times (1+i)^4 + 12000 \times \frac{(1+i)^4 - 1}{i} + 8000 = 0$$

By solving calculator,

$\therefore$ i = 36.87% > MARR, accepted

Similarly, (with the help of calculator)

For project C: i = 33.63% > MARR, accepted.

For project D: i = 37.74% > MARR, accepted.

Again, arrange the projects according to their capital investment

A, B, D and C

And Select A as a base project then

Increment	B – A	D – B	C – D
Initial investment	5000	5000	7000
Annual income	2000	2000	3000
Salvage	2500	5000	- 2000

For B – A

FW (i%) = 0

$$-5000 \times (1+i)^4 + 2000 \times \frac{(1+i)^4 - 1}{i} + 2500 = 0$$

$\therefore$ i = 32.16% > MARR So accept B and eliminate A.

Again, Incremental analysis between B and D. i.e. (D – B)

Calculating IRR for B – A

FW (i%) = 0

$$-5000 (1+i)^4 + 2000 \frac{(1+i)^4 - 1}{i} + 5000 = 0$$

$\therefore$ i = 40.00% > MARR so, select D and eliminate B.

Again, incremental analysis between C & D i.e. (C – D)

IRR for C – D

FW (i%) = 0

$$-7000 (1+i)^4 + 3000 \frac{(1+i)^4 - 1}{i} + (-2000) = 0$$

$\therefore$ i = 18.32% > MARR, So, select C & eliminate D.

Based upon IRR method analysis the best project would be project C.

Again, to select best combination:

Here, Project A, B, C & D are mutually exclusive projects.

So, Combination of Project A and Project D will be the best combination because these two have highest value of IRR as calculated earlier.

5. **Use repeatability assumption to select the best project from the following three projects.**

Project	A	B	C
Initial investment	100000	200000	250000
Annual expenditure	25000	20000	15000
Useful life	3	5	7
Salvage value	40000	50000	60000
MARR	14%	14%	14%

(2071 Bhadra)

Solution:

Here common year for A, B & C is = LCM of 3, 5 & 7 = 105 years

Study period is very large, here project A, B & C has to be repeated 35 times, 21 times and 15 times respectively.

For this case we have to use summation approach.

For Project A:

$$PW(14\%)_A = \sum_{i=0}^{34} -\frac{100000}{(1+i)^{3i}} + (-25000)\times\frac{(1+i)^{105}-1}{i}\times\frac{1}{(1+i)^{105}} + \sum_{i=1}^{35}\frac{40000}{(1+i)^{3i}}$$

↑ To convert all the all average Capital investment to year '0' year '0'

↑ to convert all annual cost to year '0'

↑ To convert value to

Now, solve by using calculator

$$PW_A = \sum_{i=0}^{34} -\frac{100000}{(1+0.14)^{3i}} + (-25000)\times\left[\frac{(1+0.14)^{105}-1}{0.14}\right]\times\frac{1}{(1+0.14)^{105}} + \sum_{i=1}^{35}\frac{40000}{(1+0.14)^{3i}}$$

$= -307665.02 - 178571.24 + 83066.05$

$= -$ Rs. 403170.21

Similarly,

For Project B:

$$PW(14\%)B = \sum_{i=0}^{20} -\frac{200000}{1.14^{5i}} + (-20000)\times\left[\frac{1.14^{105}-1}{0.14}\right]\times\frac{1}{1.14^{105}} + \sum_{i=1}^{21}\frac{50000}{1.14^{5i}}$$

$= -416118.91 - 142857.0 + 54029.78$

$= -$ Rs. 504946.13

For Project C:

$$PW(14\%)C = \sum_{i=0}^{14} -\frac{250000}{1.14^{7i}} + (-15000)\times\left[\frac{1.14^{105}-1}{0.14}\right]\times\frac{1}{1.14^{105}} + \sum_{i=1}^{15}\frac{60000}{1.14^{7i}}$$

$= -416414.52 - 107142.0 + 39939.5 = -$ Rs. 483617.02

So Based on PW, the project A would be best selection. It has lowest value of present worth (PW) = – Rs. 403170.21

6. a) Select the best project by ERR method.

Take MARR = 10% & E = 20%.

EOY	0	1	2	3	4	5	6
Project A	– 64,000	26200	29000	30,200	31,000	31,000	26,000
Project B	– 68,000	– 4,000	39,200	38,000	38,000	38,000	38,000
Project C	– 75,500	20,500	40,600	40,000	39,000	39,000	32,400

(2072 Ashwin)

Solution:

For Project A:

PW of all cost = – Rs. 64000

FW of all benefit $= 26200\times1.2^5 + 29000\times1.2^4 + 30200\times1.2^3 + 31000\times1.2^2 + 31000\times1.2^1 + 26000 =$ Rs. 285353.984

Now, $PW\times(1+i)^6 = FW$

$64000\times(1+i)^6 = 285353.984$

Solving, $i = 28.29\% >$ MARR, accepted

For Project B:

PW of all cost $= 68000 + 4000\times1.2^{-1} =$ Rs. 71333.33

FW of all benefit $= 39200\times1.2^4 + 38000\times(1.2^3+1.2^2+1.2+1)$

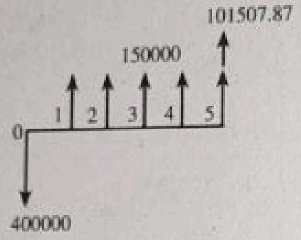
$=$ Rs. 285269.12

(6 yrs)

$$PW(10\%)\,MV = 25000\,(P/F\ 10\%, 1)$$

$$= 25000 \times \frac{1}{1.1} = Rs.\ 22727.27$$

Now, Imputed market value (IMV) = 78780.60 + 22727.27= Rs. 101507.87



We have, modified BCR = $\frac{AW(B) - AW(O\ \&\ M)}{AW(I) - AW(S)}$

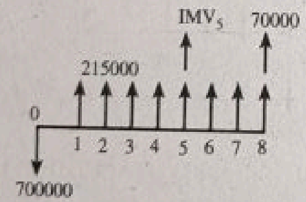
$$AW(I) - AW(S) = (400000 \times 1.1^5 - 101507.87) \times \frac{0.1}{1.1^5 - 1} = Rs.\ 88892.258$$

So, modified BCR = $\frac{150000}{88892.258} = 1.687 > 1$, ok

<u>For project B:</u>

Salvage = 10% of 700000 = Rs. 70000 Net benefit

= 250000 – 35000 = 215000



$$CR(10\%) = 700000\,(A/P, 10\%, 8) - 70000(A/F, 10\%, 8)$$

$$= 700000 \times 0.187 - 70000 \times 0.087 = Rs.\ 124810$$

$PW(10\%)_{CR}$ for remaining yrs = 124810 (P/A 10%,3)

= 124810 × 2.486 = Rs. 310384

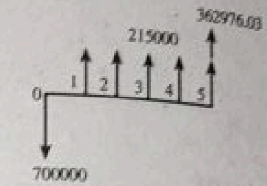
Also, $PW(10\%)_{MV}$ = 70000 (P/F,10%,3)

= 70000 × 0.7513

= Rs. 52592.03

Now, IMV = 310384 + 52592.03 = Rs. 362976.03

So,



$$AW(I) - AW(S) = (700000 \times 1.1^5 - 362976.03) \times \frac{0.1}{1.1^5 - 1}$$

= Rs. 125203.677

AW (B) – AW (O & M) = 215000

So, modified BCR = $\frac{215000}{125203.677} = 1.72 > 1$ so, ok

Now, based on modified BCR Method

BCR of B > BCR of A

So, project B would be best selection s

[Note: If more than two alternative project were given then you have to proceed similarly as for IRR & ERR) in the case of BCR also]

7. **Define mutually exclusive project, independent project and contingent project with proper combination.** (2072 Ashwin)

Solution:

Refer [2070 Magh]

8. **Compare the following two mutually exclusive projects by using (i) co-terminated (ii) Repeatability assumption taking MARR = 8%.**

	Project A	Project B
Initial cost	1, 50, 000	2, 00, 000
Annual revenue	90, 000	1, 00, 000
Operating cost	20, 000	20, 000
Life year	4	6
Salvage value	80, 000	1, 20, 000

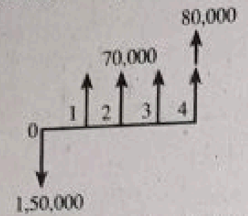
(2073 Bhadra)

Solution:

i) By using co-terminated

For Project A:

Net annual revenue = Revenue - expenses = 90000 - 20000 = 70000



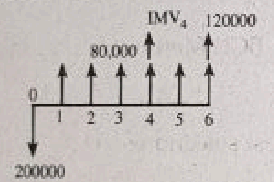
$$FW(8\%)_A = -150000\,(F/P, 8\%, 4) + 70{,}000\,(F/A, 8\%, 4) + 80000$$

$$= -150000\,(1 + 0.08)^4 + 70000\left(\frac{1.08^4 - 1}{0.08}\right) + 80000$$

$$= \text{Rs. } 191354.49$$

For alternative B:

Net annual revenue = 100000 - 20000 = 80000



By using IMV technique

Capital recovery,

$$CR(8\%) = 200000\,(A/P, 8\%, 6) - 120000\,(A/F, 8\%, 6)$$

$$= 200000 \times 1.08^6 \times \frac{0.08}{1.08^6 - 1} - 120000 \times \frac{0.08}{1.08^6 - 1}$$

$$= 43263.07 - 16357.85 = \text{Rs. } 26905.22$$

Now, PW at the end of 4th year for remaining 2 years.

$$PW(8\%)_{CR} = 26905.22\,(P/A, 8\%, 2)$$

$$= 26905.22 \times \left(\frac{1.08^2 - 1}{0.08}\right) \times \frac{1}{1.08^2} = \text{Rs. } 47979.13$$

Again, PW at the end of 4th year or original market value at the end of useful life (6 years)

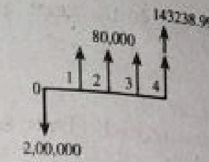
i.e. at the end of 4th year of salvage value (Rs. 120,000)

$$\text{So, } PW(8\%)\,MV = 120000\,(P/F, 8\%, 2)$$

$$= 120000 \times (1 + 0.08)^3 = \text{Rs. } 95259.87$$

Now, imputed marked value (IMV) = 47979.13 + 95259.86

= 143238.99



$$FW(8\%)_B = -200000\,(F/P, 8\%, 4) + 80000\,(F/A, 8\%, 4) + 143238.99$$

$$= -200000 \times (1 + 0.08)^4 + 80000\left[\frac{(1 + 0.08)^4 - 1}{0.08}\right] + 143238.99$$

$$= -272097.79 + 360488.96 + 143238.89 = \text{Rs. } 231630.16$$

Therefore, based on co-terminated assumption FW for project B is higher. (Rs. 231630.16).

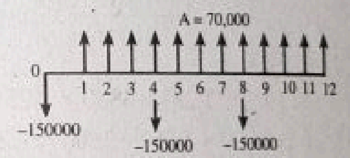
So, project B is best selection.

iii) **By using repeatability method**

The LCM of useful lives of alternatives A & B is 12 years, using the repeatability assumptions and a 12 years study period.

For Project A:

Net revenue = 70,000

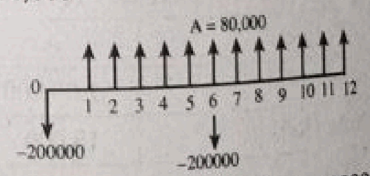


$$\text{Now, } PW(8\%)_A = -150000 - 15000\,[(P/F, 8\%, 4) + (P/F, 8\%, 8)] + 70000\,(P/A, 8\%, 12)$$

$$= -150000 - 150000\,[(1 + 0.08)^{-4} + (1 + 0.08)^{-8}] + 70000 \times \left[\frac{1.08^{12} - 1}{0.08}\right] \times \frac{1}{1.08^{12}} = \text{Rs. } 186230.65$$

For Project B:

Net revenue = 80,000



$$PW(8\%)_B = -200000 - 200000\,(P/F, 8\%, 6) + 80000\,(P/A, 8\%, 12)$$

$$= \text{Rs. } 276852.316$$

Here PW of B > PW of A

So, Project B would be best selection.

Now, PW $(1 + i)^6$ = FW

$71333.33 (1 + i)^6$ = 285269.12

∴ I = 25.99% > MARR, accepted.

For Project C:

PW of all cost = Rs. 75,500

FW of all benefit = $20500 \times 1.2^5 + 40600 \times 1.2^4 + 40000 \times 1.2^3 + 39000 \times (1.2^2 + 1.2) + 32400$ = Rs. 339678.72

Now, PW $(1 + i)^6$ = FW

$75500 \times (1 + i)^6$ = 339678.72

∴ i = 28.48% > MARR, accepted

Again, select project A as the base project and arrangement of project according to initial investment is A, B & C.

Incremental analysis between A & B i.e. (B – A)

Increment	0	1	2	3	4	5	6
B – A	– 4000	– 30200	10200	7800	7000	7000	12000
C – B	– 7500	24500	1400	2000	1000	1000	– 5600

For (B – A)

PW of all cost $= -4000 - \frac{30200}{1.2}$ = Rs. – 29166.67

FW of all benefit = $10200 \times 1.2^4 + 7800 \times 1.2^3 + 7000 (1.2^2 + 1.2) + 12000$
= Rs. 65109.12

Now, PW $(1 + i)^6$ = FW

$29166.67 (1 + i)^6$ = 65109.12

∴ i = 14.32 > MARR so accept B, eliminate A

Again, Incremental analysis between B and C i.e. (C – B)

For C – B:

PW of all cost $= -7500 - \frac{5600}{1.2^6}$ = Rs. – 9375.43

FW of all benefit = $24500 \times 1.2^5 + 1400 \times 1.2^4 + 2000 \times 1.2^3 + 1000 \times (1.2^2 + 1.2)$ = Rs. 83767.106

Now, PW $(1 + i)^6$ = FW

$9375.43 (1 + i)^6$ = 83767.106

∴ I = 44% > MARR Select C & Eliminate B

So, Based on ERR method, the project C would be best selection.

b) **Co-terminating both project at 5 years and select the best project by modified BCR (using AW formulation). Take salvage value of each project = 10% of first cost. MARR = 15%**

Project	First cost	Annual Benefits	Annual O & M costs	Useful life
A	Rs. 400000	Rs. 175,000	Rs. 25000	6 yrs
B	Rs. 700000	Rs. 250,000	Rs. 35000	8 yrs

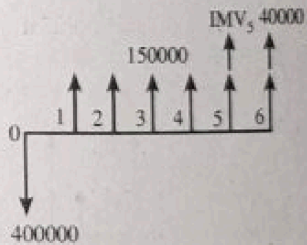
(2072 Ashwin)

Solution:

Here, for both project study period is lesser than the useful life. So, we have to use imputed market value (IMV) technique.

For project A: Salvage = 10% of 4000000 = 40000 &

Net benefit = 175000 – 25000 = 150000



Capital recovery CR (10%) = 400000 (A/P, 10%,6) – 40000 (A/F, 10%,6)

$= 400000 \times 1.1^6 \times \frac{0.1}{1.1^6 - 1} - 40000 \times \frac{0.1}{1.1^6 - 1}$

= Rs. 86658.65

PW at the end of 5^{th} year for remaining years

PW (10%) $_{CR}$ = 86658.65 (P/A, 10%, 1)

$= 86658.65 \times \frac{1}{1.1^1}$ = Rs. 78780.60

PW at the end of 5^{th} year of original market value at the end of useful life

9. **Define mutually exclusive, contingent and independent projects with suitable example.** (2073 Bhadra)

Solution: *Refer 2070 (Magh)*

10. **Select the best proposal using ERR (Take ε - 25% and MARR = 20%)**

EOY	0	1	2	3	4	5	6
Proposal A	- 6000	2400	2800	2900	3000	3000	2450
Proposal B	-7000	1900	3800	3700	3600	3600	3300

(2073 Magh)

Solution:

<u>For Proposal A</u>

PW of all cost = 6000

FW of all benefit = $2400 \times 1.25^5 + 2800 * 1.25^4 + 2900 * 1.25^3 + 3000 * 1.25^2 + 3000 * 1.25^1 + 2450 = 30711.72$

Now, $PW (1 + i)^6 = FW$

$6000 (1 + i)^6 = 30711.71$

Solving, i = 31.27% > MARR, accepted

<u>For Proposal B,</u>

PW of all cost = 7000

FW of all benefit = $1900 \times 1.25^6 + 3800 \times 1.25^4 + 3700 \times 1.25^3 + 3600 \times 1.25^2 + 3600 \times 1.25 + 3300 = 35727.24$

Now, $PW (1 + i)^6 = FW$

$7000 (1 + i) = 3572.24$

Solving, we get, i = 31.21% > MARR accepted

Based on ERR method, proposal A should be selected because it has a higher internal Rate of Return (i)

11. **What do you mean by mutually exclusive, contingent and independent project? Compute the following projects by using repeatability assumption when NARR is 10%.**

Project	A	B
Initial Investment (Rs)	1,00,000	1,50,000
Annual Revenue (Rs)	15,000	20,000
Annual Cost (Rs)	3,000	4,000
Useful life (years)	6	8
Salvage value (Rs.)	5,000	10,000

(2073 Magh)

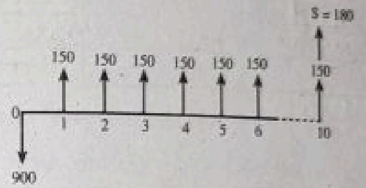
Solution: *Refer "2070 Magh" Q.No. 1*

12. **Choose the best project among these alternatives using IRR. If MARR = 15% and study period is 10 years. Salvage value is 20%.**

Project	A	B	C	D
First cost Rs.	900	1500	2500	4000
Annual Revenue Rs.	150	276	400	925

(2074 Bhadra)

Solution: For Project A



Calculating IRR

PW (i%) = 0

$$-900 + 150\left[\frac{(1+i)^{10}-1}{i(1+i)^{10}}\right] + \frac{180}{(1+i)^{10}} = 0$$

On solving, i = 0.121 = 12%

∴ IRR of A = 12%

Similarly for Project B

PW (i%) = 0

$$-1500 + 276\left[\frac{(1+i)^{10}-1}{i(1+i)^{10}}\right] + \frac{300}{(1+i)^{10}} = 0$$

On solving, i = 0.143

∴ IRR of B = 14.30%

Again for C, PW (i %) = 0

$$-2500 + 400\left[\frac{(1+i)^{10}-1}{i(1+i)^{10}}\right] + \frac{500}{(1+i)^{10}} = 0$$

On solving, i = 0.1128 ∴ IRR of C = 11.28%

Again for D, PW (i %) = 0

$$-4000 + 925\left[\frac{(1+i)^{10}-1}{i(1+i)^{10}}\right] + \frac{800}{(1+i)^{10}} = 0$$

On solving

i = 0.20 = 20%

∴ IRR of D = 20%

So the best project among given alternative is Project D with maximum IRR of 20%.

13. Consider the following two mutually exclusive alternatives; recommend the best alternatives using repeatability assumptions. MARR = 15%

	Project (A)	Project (B)
Initial cost	100,000	150,000
Annual cost	25,000	12,000
Salvage value	40000	50000
Useful life	6 yrs	10 yrs

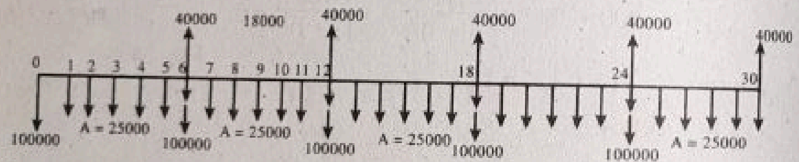
(2074 Bhadra)

Solution:

From given data above using repeatability assumptions:

Here common time period for 6 and 10 year is 30 years.

For Project A:



Now using cash flow diagram and MARR = 15%

$FW(15\%)_{30} = -100000\,[(F/P,15\%,30) + (F/P,15\%,24) + (F/P,15\%,18) + (F/P,15\%,12) + (F/P,15\%,6)]$

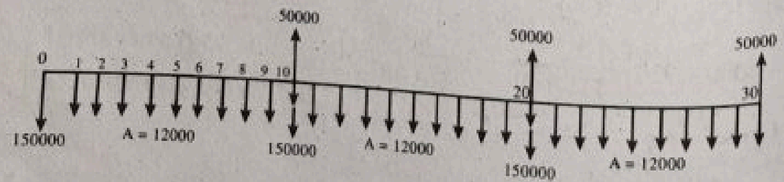
$+ 40000\,(F/P,15\%,24) + (F/P,15\%,18) + (F/P,15\%,12) + (F/P,15\%,6) + 1] - 25000\,(F/A,15\%,30)$

$= -100000\,[1.15^{30} + 1.15^{24} + 1.15^{18} + 1.15^{12} + 1.15^{6}]$

$+ 40000\,[1.15^{24} + 1.15^{18} + 1.15^{12} + 1.15^{6} + 1] - 25000 \times \dfrac{1.15^{30} - 1}{0.15}$

$= -11087947 + 1986557.62 - 10868628.66 = \text{Rs.} -20369618.04$

For Project B:



$FW(15\%)_{30} = -150000\,[1.15^{30} + 1.15^{20} + 1.15^{10}] + 50000\,[1.15^{20} + 1.15^{10} + 1]$

$- 18000 \times \dfrac{1.15^{30} - 1}{0.15}$

$= -12993580.06 + 1070604.75 - 5216941.75 = \text{Rs.} -17139917.07$

Here FW of B > FW of A. So, Project could be the best solution.

14. Compare following two projects by IRR method when i = 10% per year.

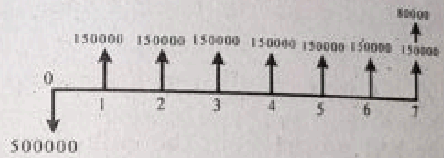
	Initial cost	Annual revenue	Annual cost	Salvage value	life year
Project A	500000	200000	50000	80000	7
Project B	700000	300000	100000	150000	7

(2075 Baisakh)

Solution: Here the cash flow diagram is,

For Project A Net revenue annually (A) = Revenue – cost

= 200000 – 50000 = 150000



Further proceed as QN 1 of (2074 Bhadra) and select project with higher value of IRR among A and B for implementation.

15. Select best project by repeatability assumption when MARR = 13%

	Initial cost	Annual revenue	O & M	Life year	Salvage
Project X	400000	175000	50000	4	100000
Project Y	700000	250000	70000	6	150000

(2075 Baisakh)

Solution: For this Net revenue = Annual revenue – O & M cost

i.e. for project X (A) = 175000 – 50000 = 125000

& for project Y (A) = 250000 – 70000 = 180000

And common time period for 4 & 6 years is 12 years

Taking 12 years of total service life.

Use repeatability method as Q.N. 2 of (2074 Bhadra) and choose project with higher value of FW.

16. Define independent and contingent projects. Find present worth from annual cash flow series of Rs. 5,000 forever when i = 8% per year. **(2075 Baisakh)**

Solution: Refer chapter for definition.

Given, A = 5000

Time period = forever

i = 8%

It is the case of continuous compounding and continuous cash flow.

So that, $P = A "\{(e^{rN} - 1)/re^{rN}\} = 5000\ e^{0.08}$

17. a. Recommend the best project from the following two projects taking study period as 6 years. Assume MARR = 10% per year.

Project	A	B
Investment	350,000	500,000
Annual Revenue	130,000	175,000
Annual Cost	15,000	25,000
Salvage Value	35,000	50,000
Useful life	5 years	8 years

(2076 Bhadra)

Solution: *Refer solution of (2072 Ashwin)*

b) Recommend the best project from the following cash flow of a mutually exclusive

Projects using modified benefit cost ratio method. MARR = 10%

Project	A	B
Initial Investment	24,00,000	35,50,000
Annual Revenue	8,20,000	12,00,000
Annual Cost	1,10,000	1,40,000
Salvage Value	2,25,000	3,50,000
Useful life	10 years	10 years

Solution: *Refer solution of (2071 Magh)*

5 REPLACEMENT ANALYSIS

Organizations providing goods/services use several facilities like equipment and machinery which are directly required in their operations. In addition to these facilities, there are several other items which are necessary to facilitate the functioning of organizations.

All such facilities should be continuously monitored for their efficient functioning; otherwise, the quality of service will be poor. Besides the quality of service of the facilities, the cost of their operation and maintenance would increase with the passage of time. Hence, it is an absolute necessity to maintain the equipment in good operating conditions with economical cost. Thus, we need an integrated approach to minimize the cost of maintenance. In certain cases, the equipment will be obsolete over a period of time. If a firm wants to be in the same business competitively, it has to take decision on whether to replace the old equipment or to retain it by taking the cost of maintenance and operation into account.

There are two basic reasons for considering the replacement of an equipment—physical impairment of the various parts or obsolescence of the equipment. Physical impairment refers only to changes in the physical condition of the machine itself. This would lead to a decline in the value of the service rendered, increased operating cost, increased maintenance cost or a combination of these. Obsolescence is due to improvement of the tools of production, mainly improvement in technology. So, it would be uneconomical to continue production with the same machine under any of the above situations. Hence, the machines are to be periodically replaced.

Sometimes, the capacity of existing facilities may be inadequate to meet the current demand. Under such situation, the following alternatives will be considered.

- Replacement of the existing equipment with a new one.
- Augmenting the existing one with an additional equipment.

Types of Replacement Problem

Replacement study can be classified into two categories:

(a) Replacement of assets that deteriorate with time (Replacement due to gradual failure, or wear and tear of the components of the machines).

This can be further classified into the following types:

(i) Determination of economic life of an asset.

(ii) Replacement of an existing asset with a new asset.

(b) Simple probabilistic model for assets which fail completely (replacement due to sudden failure).

5.1 Fundamentals of Replacement Analysis

5.1.1 Basic concepts and Terminology

Replacement analysis is a choice between the present asset (called defender) and new alternatives (called challenger).

Concepts of Replacement analysis refers to selection of similar but new assets to replace the existing assets to meet current and future requirement more economically.

Replacement projects are decision problems involving the replacement of existing obsolete or worn-out assets. The continuation of operations is dependent on these assets. The failure to make an appropriate decision results in a slowdown or shutdown of the operations. The question is when existing equipment should be replaced with more efficient equipment.

This situation has given rise to the use of the terms **defender** and **challenger,** terms commonly used in the boxing world. In every boxing class, the current defending champion is constantly faced with a new challenger. In replacement analysis, the defender is the existing machine (or system), and the challenger is the best available replacement equipment.

An existing piece of equipment will be removed at some future time, either when the task it performs is no longer necessary or when the task can be performed more efficiently by newer and better equipment. The question is not *whether* the existing piece of equipment will be removed, but *when* it will be removed. A variation of this question is why we should replace existing equipment at the current time, rather than postponing replacement of the equipment by repairing or overhauling it.

Another aspect of the defender-challenger comparison concerns deciding exactly which equipment is the best challenger. If the defender is to be replaced by the challenger, we would generally want to install the very best of the possible alternatives.

Reasons for Replacement:

1. Physical impairment (Deterioration): These are changes that occur in the physical condition of the asset. It includes wear and tear of equipment. Normally aging result, in the operation of an asset becoming less efficient.

2. Inadequacy: Capital assets are used to produce goods and services that satisfy human wants. When the demand for a service is increases then existing asset does not have sufficient capacity to fill that.

3. Obsolescence: Due to impact of changes in technology result in more frequent replacement of existing assets with new and better challengers.

4. Financing: Financial factors involve economic opportunity changes external to the physical operation. For example, the rental (lease) of assets may become more attractive than ownership.

Some definition:

- **Economic service life:** Economic service life of assets is defined as the period of time that results in the minimum equivalent uniform annual cost of owning and operating an asset.
- **Ownership life:** It is the period between the date of acquisition and the date of disposal by a specific owner.
- **Physical Life:** It is the period between original acquisition and final disposal of an asset over its succession of owners.
- **Useful life:** It is the time period that an asset is kept in productive service. It is the estimate of how long an asset is expected to be used in business to produce income.
- **Sink cost:** A sink cost is money that has already been spent and cannot be recovered. Sink costs are also called retrospective costs.

5.1.2 Approaches for comparing defender and challenger

Two basic approaches to analyzing replacement problems are the **cash-flow approach** and the **opportunity-cost approach.**

1. **Cash flow approach:** In this approach, proceeding from the sale of

defender and is treated as down payment toward purchase of challenger. This approach explicitly considers the actual cash-flow consequences for each replacement alternative as it occurs. Typically, the net proceeds from the sale of the defender are subtracted from the purchase price of the challenger.

2. **Opportunity cost approach:** In this approach, proceeding from sale of defender and is treated as investment required to keep defender itself.

This approach views the net proceeds from the sale of the defender as an opportunity cost of keeping the defender. That is, instead of deducting the salvage value from the purchase cost of the challenger, we consider the salvage value an investment required in order to keep the asset.

- Basic opportunity cost approach is considered for comparing defender and challenger. Replacement analysis is carried out comparing annual equivalent cost (AEC) of defender and challenger.
- After comparison, the option having lower value of annual equivalent cost is accepted. Where AEC is calculated by considering salvage value of an asset (if any) with the help of annual worth (AW) of all cost throughout its life.
- The total annual equivalent cost (AEC) of owning & operating an Asset (AEC) (i %) is the summation of the capital recovery cost and annual equivalent of operating cost of an assets.

I.e. AEC (i %) = CR (i %) + OC (i %)

5.2 Economic Service Life of Challenger and Defender

Economic service life: The economic service life (ESL) is the number of year which minimizes the equivalent uniform annual cost of owning and operating an asset, and is often shorter than the useful or physical life.

Capital Recovery cost (CR) = I (A/P, i %, N) – S (A/F, i %, N)

$$\text{Operating cost (OC)} = \sum_{n=1}^{N} OC_n\,(P/F, i\,\%, n)\,(A/P, i\%, N)$$

Total cost = (CR + OC)

Economic service life of challenger: The economic life of an assets

minimizes the equivalent uniform annual cost of owning and operating an asset, and is often shorter than the useful or physical life. It is essential to know a challenger's economic life in view of the principle that new and existing assets should be compared over their economic lives.

Economic service life of the defender: The economic life of the defender is often one year. Generally defender involves a lower initial cost than purchasing challenger but it requires more annual operating (repair & maintenance also) cost due to old technology and continuing use (aging).

Capital Recovery with Return: Consider the following data of a machine. Let P = purchase price of the machine, F = salvage value of the machine at the end of machine life, n = life of the machine in years, and i = interest rate, compounded annually. The corresponding cash flow diagram is shown in Fig. here.

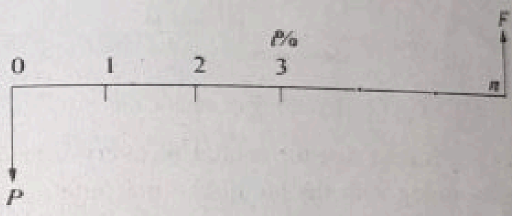


Fig: Cash flow diagram of machine

The equation for the annual equivalent amount for the above cash flow diagram is

AE (i) = (P – F) x (A/P, i, n) + F x i

This equation represents the capital recovery with return.

Determination of Economic Life of an Asset

Any asset will have the following cost components: Capital recovery cost (average first cost), computed from the first cost (purchase price) of the machine. Average operating and maintenance cost (O & M cost) Total cost which is the sum of capital recovery cost (average first cost) and average maintenance cost. A typical shape of each of the above costs with respect to life of the machine is shown in Fig. below

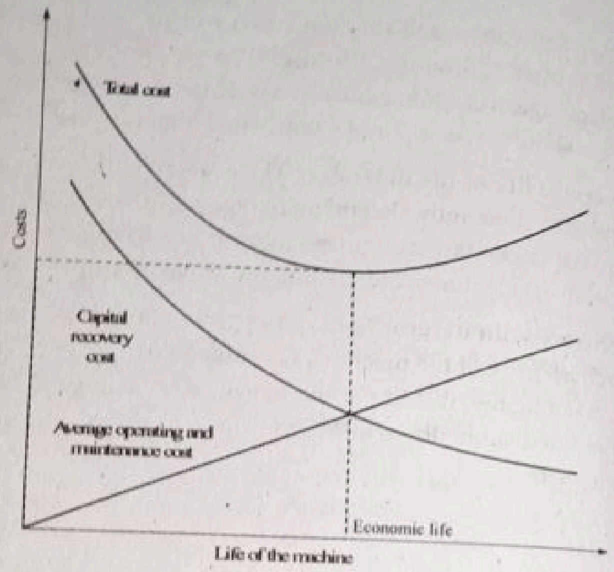


Fig: Chart showing economic life

From Fig. above, it is clear that the capital recovery cost (average first cost) goes on decreasing with the life of the machine and the average operating and maintenance cost goes on increasing with the life of the machine. From the beginning, the total cost continues to decrease up to a particular life and then it starts increasing. The point where the total cost is minimum is called the economic life of the machine.

If the interest rate is more than zero per cent, then we use interest formulas to determine the economic life. The replacement alternatives can be evaluated based on the present worth criterion and annual equivalent criterion.

Example: Determine the choice between a defender that has a current market value of Rs. 25000 and challenger that can be purchased for 37500. Both have a same life of 3 years with salvage value zero at the end of life. Take MARR = 12%. Their operating costs (i.e. Cash flow) values are gives as:

EOY	Defender (D)	Challenger (C)
0	25000	37500
1	8500	2500
2	10000	5500
3	12500	6500

Solution:

Determine annual equivalent cost for both defender and challenger

For Defender (D)

The market value of 25000 is considered as capital investment, as per opportunity cost approach

Then,

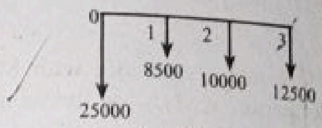


Fig.: CFD of defender (D)

$AEC (12\%)_D$ = [-25000 (F/P,12%,3) - 8500 (F/P, 12%2) - 10000 (F/P,12%,1) - 12500] × (A/F, 12%, 3)

= $[-25000 (1 + 0.12)^3 - 8500 (1 + 0.12)^2 - 10000 (1 + 0.12)$ - 12500] (A/F, 12%, 3)

= - 49460 (A/F,12%,3)

$= -69485.60 \times \frac{0.12}{1.12^3 - 1}$

= Rs. - 20591.98

For challenger (C)

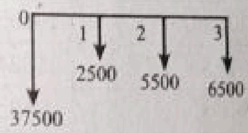


Fig.: CFD of challanger (C)

$AEC (12\%)_C$ = [-37500 (F/P,12%,3) - 2500 (F/P, 12%2)- 5500 (F/P,12%,1

- 6500] × (A/F, 12%, 3)

$= -68564.19 \times \frac{0.12}{1.12^3 - 1}$

= Rs. - 20295

Hence, $AEC (12\%)_D > AEC (12\%)_C$

So replacement should be made.

Example

One electric company has decided to replace existing generator with a new one having initial cost of 54000 with useful life of 8 years and having operating cost of Rs. 9000 in the first year. For the remaining year operating cost increases by 15% over the previous year's operating cost & salvage value declines each year by 20% from the previous year's salvage value. Determine the economic service life of this new machine by considering MARR = 12% per year.

Solution:

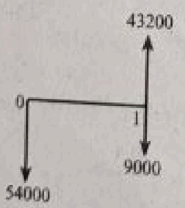
Here to determine economic service life with MARR = 12%, for given cash flow information, we can use the following equation of AEC

Annual equivalent cost AEC (i %) = CR (i %) + OC (i %)

Now,

N	Operation cost	Salvage cost
0	-	54000
1	9000	43200
2	10350	34560
3	11902.50	27648
4	13687.87	22118.4
5	15741.05	17694.72
6	18102.21	14155.77
7	20817.5	11324.62
8	23940.8	9059.70

For year N = 1;



$$AEC\ (12\%)_1 = CR\ (12\%) + OC\ (12\%)$$

$$= [54000\ (F/P,12\%,1) - 43200 + 9000] \times (A/F,\ 12\%,\ 1)$$

$$= (54000 \times 1.12 - 43200 + 9000) \times \frac{0.12}{1.12^1 - 1} = \text{Rs. } 26280$$

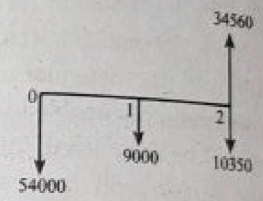
Note: We can also calculate CR & OC separately as,

$$CR\ (12\%) = 54000\ (A/P,12\%,1) - 43200\ (A/F,12\%,1) = \text{Rs. } 17280$$

$$OC\ (12\%) = 9000$$

Then AEC (12%) = 17280 + 9000 = Rs. 26280

<u>For year N = 2</u>



$$AEC\ (12\%)_2 = [54000\ (F/P,12\%,2) - 34560 + 9000\ (F/P,12\%,1) + 10350] \times (A/F,12\%,2)$$

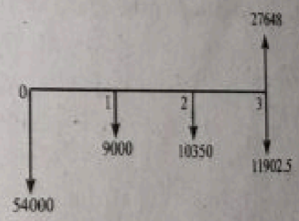
$$= (54000\ (1.12)2 - 34560 + 9000\ (1.12) + 10350) \times (A/F,\ 12\%,2)$$

$$= 53607.6 \times \frac{0.12}{1.12^2 - 1}$$

$$= \text{Rs. } 25287$$

Similarly,

For N = 3



$AEC\ (12\%)_3$ = Rs. 24597

& For N = 4; $AEC\ (12\%)_4$ = Rs. 24165

For N = 5; $AEC\ (12\%)_5$ = Rs. 23955

For N = 6; $AEC\ (12\%)_6$ = Rs. 23931

For N = 7; $AEC\ (12\%)_7$ = Rs. 24072

For N = 8; $AEC\ (12\%)_8$ = Rs. 24354

So, From above calculation AEC is decreases from N = 1 to 6, then after then it start to increase.

So, from analysis, economic service life (N) = 6 years even though, it has useful life = 8 yrs.

5.3 Replacement Analysis when Required Service Life is long

5.3.1 Requirement Assumption & decision Framework:

There are three assumption to determine the optimal time to replace the defender. They are

a) Planning horizon (study period) b) Technology

c) Relevant cash flow information

(a) **Planning horizon:** By the planning horizon, we simply mean the service period required by the defender and a sequence of future challengers. When we are simply unable to predict when the activity under consideration will be terminated, infinite planning horizon is used. But when the project will have a definite & predictable duration, the replacement policy should be formulated more realistically on the basis of a finite planning horizon.

(b) **Technology:** Predictions of technological patterns over the planning horizon refer to the development of types of challengers that may replace those under study. A number of possibilities exist in predicting purchase cost, salvage value and operating cost that are dictated by the efficiency of new machine over the life of an existing asset.

If we assume that all future machines will be the same as those now in service, we are implicitly saying that no technological progress in the area will occur. In other cases, we may explicitly recognize the possibility of machines becoming available in the future that will be significantly more efficient, reliable, or productive than those currently on the market. (Personal computers are a good example.) This situation leads to the recognition of technological change and obsolescence. Clearly, if the best available machine gets better and better over time, we should certainly investigate the possibility of delaying an asset's replacement for a couple of years—a viewpoint that contrasts with the situation in which technological change is unlikely.

(c) **Revenue and cost patterns over the life of an Asset:** Many varieties of predictions can be used to estimate the patterns of revenue, cost of salvage value over the life of an asset. Sometimes revenue is

constant, but assets increase, with salvage value decreases, over the life of a machine. In other situation, a decline in revenue over the life of piece of equipment can be expected. The specific situation will determine whether replacement analysis is directed towards cost minimization or profit maximization. We formulate a replacement policy for an asset whose salvage value does not increase with age.

Decision framework:

Decision framework is developed for indicating a replacement sequence of an assets with notation (J_0,n_0) (J_1,n_1) (J_2,n_2)........(J_k,n_k) where such pair (J,n) indicates a types of asset and life time over which that asset will be retained.

Normally in the case of defender and challenger

Where, $J_0 \rightarrow$ defender

$J \rightarrow$ Challenger

If defender asset (J_0) replaced now then $n_0 = 0$ & it can be written as $(J_0, 0)$

The sequence $(J_0,2)$, $(J_1,5)$ $(J_2,3)$ indicates retaining the defender for two years and then replacing defender with an asset type J_1 (Challenger - I) & it is used for next 5 years & then again replacing J_1 with an asset type J_2 (Challenger– II) & using it for three years. Hence total planning horizon covers $2 + 5 + 3 = 10$ yrs.

Decision Criterion

Although the economic life of the defender is defined as the additional number of years of service which minimizes the annual equivalent cost (or maximizes the annual equivalent revenue), that is not necessarily the *optimal* time to replace the defender. The correct replacement time depends on data on the challenger, as well as on data on the defender.

As a decision criterion, the Annual Equivalent (AE) method provides a more direct solution when the planning horizon is infinite. When the planning horizon is finite, the PW method is more convenient to use. We will develop the replacement decision procedure for both situations. We begin by analyzing an infinite planning horizon without technological change. Even though a simplified situation such as this is not likely to occur in real life, the analysis of this replacement situation introduces methods that will be useful in analyzing infinite-horizon replacement problems with technological change.

5.3.2 Replacement Analysis under infinite planning horizon

Based on the infinite planning horizon, the service is required for a very long time. Either we continue to use the defender to provide the service, or we replace the defender with best available challenger for the same service requirement.

Procedure:

Step 1: Compute economic life for defender say N_D^* and also compute economic life for challenger say N_C^*. The corresponding annual equivalent cost is AEC_D^* and AEC_C^*.

Step 2: Compare AEC_D^* and AEC_C^*.

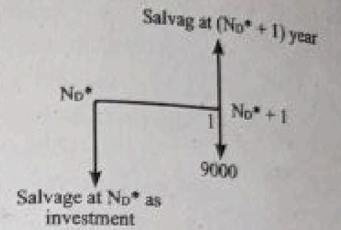
If $AEC_D^* > AEC_C^*$, Accept challenger, it is more costly to keep the defender than to replace it with the challenger. Thus, the challenger should replace the defender now.

If $AEC_D^* < AEC_C^*$, it cost less to keep defender than replace it by challenger. Thus, the defender should *not* be replaced now. The defender should continue to be used at least for the duration of its economic life if there are no technological changes over that life.

I.e. No replace on the year of (N_D^*)

Step 3: If the defender should not be replaced now, when should it be replaced? First we need to continue to use it until its economic life is over. Then we should calculate the cost of running the defender for one more year after its economic life. If this cost is greater than AEC_C^* the defender should be replaced at the end of its economic life. Otherwise, we should calculate the cost of running the defender for the second year after its economic life. If this cost is bigger than AEC_C^* the defender should be replaced one year after its economic life. The process should be continued until we find the optimal replacement time. This approach is called **marginal analysis**; that is, we calculate the incremental cost of operating the defender for just one more year. In other words, we want to see whether the cost of extending the use of the defender for an additional year exceeds the savings resulting from delaying the purchase of the challenger. Here, we have assumed that the best available challenger does not change

Then compute the AEC for defender after N_D^* year i.e. for (N_D^* + 1) year



[As, OC = Operational Cost]

Then compute AEC (i %) & if AEC (i %) > AEC_C^* then replaced. Otherwise check for next (N_D^* + 2) year.

Example:

Old - machine: Current salvage value = Rs 5,000 decreasing at annual rate of 25 % over previous year.

Required overhaul = Rs 1, 200 to be used for 6 years

Operating cost = Rs, 2,000 for first year increasing by Rs 1,500 per year.

New - Machine: I = 10,000

Salvage value = 6,000 other 1 year and will decline by 15% each year.

Operating cost = Rs 2, 200 in first year increasing by 20% year thereafter.

For economic life determine when the old machine should be replaced at MARR = 15 %

Solution: Economic service life of defender:

Initial cost = Rs 5,000 + Rs 1, 200= Rs 6, 200

n	Operating cost	Salvage value
0	0	5000
1	2000	3750
2	3500	2813
3	5000	2109
4	6500	1582
5	8000	1187
6	9500	890

At n = 1,

AEC_D = 6, 200(A/P, 15%,1) + 2, 000 + 1,500 (A/G, 15%,1) - 3750 (A/F 15%,1)

= 6, 200 x (1.15) + 2, 000 + 1,500 x (o) - 3750 x (1)

= 5, 380

At n = 2,

AEC_D = 6, 200(A/P, 15%, 2) + 2,000+1,500(A/G, 15%, 2) - 2813 (A/P, 15%, 2)

= 5, 203

Similar,

At n = 3, AEC_D = 5469

At n = 4, AEC_D = 5844

At n = 5, AEC_D = 6258

At n = 6, AEC_D = 6682

Hence, economic service life of defender is 2 years and corresponding AEC_D Rs, 5,203.

Economic Servile life of challenger:

n	market value	Operating cost
0	10,000	0
1	6,000	2200
2	5100	2640
3	4335	3168
4	3685	3802
5	3132	4562
6	2662	5474

At n = 1

AEC_c = 10,000(A/P, 15%, 1) – 6,000(A/F, 15%, 1) + 2,200(A/F, 15% 1)

= 10,000(1.15) – 6, 000(1) + 2200(1)

= 7,700

A + n = 2,

AEC_c = 10,000(A/P, 15%, 2) – 5100 (A/F, 15%, 2) + 2,200 (R/F,15%, 2) (A/P, 15% 2) + 2,640(A/F, 15% 2)

= 6,184

At n = 3, AEc = 5,756

At n = 4, AEC_c = 5, 625

At n = 5 , AEC_c = 5721

Hence, economic service life of challenger is 4 years and corresponding AEC_c = 5,625

Now,

At ESL,

$AEC_D < AEC_C$

i.e. defender should not be replaced now.

To know when defender should be replaced, we should perform marginal analysis.

Opportunity cost at end of year 2 = Rs 2,813

Operating cost for 3rd year = Rs 5,000

Salvage value at end or 3rd year = Rs, 2,109

∴ Equivalent cost of reusing defender for one more year. = 2813 (F/p, 15%, 1) + 5000 – 2109

= 6126 (Which is greater than AEC_C)

i.e. Defender should be replaced at end of year.

5.3.3 Replacement Strategies under the Finite Planning Horizon

If the planning period is finite (for example, eight years), a comparison based on the AE method over a defender's economic service life does not generally apply. The procedure for solving such a problem with a finite planning horizon is to establish all "reasonable" replacement patterns and then use the PW values for the planning period to select the most economical pattern.

Example: A hydropower company has a contract to provide electricity to government of Nepal for the next 8 years. It can produce hydroelectricity using old turbine or the newly bought turbine. Consider old turbine as defender & new turbine as challenger and by using annual equivalent costs technique, compute their economic service life? And what is the best replacement strategy?

Number of years (N)	Annual equivalent cost at MARR = 10%	
	Old turbine	New turbine
1	2575000	3750000
2	2550000	3075000
3	2750000	2930000
4	2980000	2915000
5	3220000	2950000

Solution:

From inspection

For old turbine (defender):

Economic service life = 2nd year (year at minimum value of AEC)

For New turbine (challenger):

Economic service life = 4th year (year at minimum value of AEC)

Now, Likely replacement patterns under finite planning horizon of
years be

Option 1: (J_0, 0) (J, 4) (J, 4)

Option 2: (J_0, 1) (J, 4) (J, 3)

Option 3: (J_0, 2) (J, 4) (J, 2)

Option 4: (J_0, 3) (J, 4) (J, 1)

Option 5: (J_0, 4) (J, 4)

Option 1: (J_0, 4) (J, 4)

Here, $J_0 \rightarrow$ Defender $\qquad$ $J \rightarrow$ Challenger

Note: Here we prepared six options but result of best strategies always lies on where for both J_0 & J are within the limit of their economic life

I.e. most likely strategies are

option 1: (J_0,0) (J,4)(J,4), option 2: (J_0,1) (J,4)(J,3) & option 3: (J_0,2) (J,4)(J,3)

For option 1: (J_0, 0) (J, 4) (J, 4)

$$PW(10\%)_1 = 2915000 (P/A,10\%, 8)$$

$$= 291500\left(\frac{1.1^8 - 1}{0.1}\right) \times \frac{1}{1.1^8} = 2915000 \times 5.33 = \text{Rs. } 15551309.8$$

For option 2: (J_0, 1) (J, 4) (J, 3)

$$PW(10\%)_2 = 2575000 (P/A,10\%,1) + [2915000 (P/A,10\%4)] (P/F,10\%,1) + [2930000 (P/A, 10\%;3)] (P/F,10\%,5)$$

$$= 2575000\left[\frac{1.1^1 - 1}{0.1}\right] \times \frac{1}{1.1} + 2915000\left[\frac{1.1^4 - 1}{0.1}\right] \times \frac{1}{1.1^4} \times \frac{1}{1.1} + 2930000 \times \frac{1}{1.1^3} \times \frac{1}{1.1^5}$$

$$= 2340932.5 + 8400320 + 4524260 = \text{Rs. } 15265512.50$$

For option 3: (J_0, 2) (J, 4) (J, 2)

$$PW(10\%)_3 = 2550000 (P/A,10\%,2) + [2915000 (P/A,10\% 4)] (P/F,10\%,2) + [3075000 (P/A, 10\%,2)] (P/F,10\%,6)$$

$$= 4425525 + 7636150 + 3012545 = \text{Rs. } 15074220$$

For option 4: (J_0, 3/) (J, 5)

$$PW(10\%)_4 = 2750000 (P/A,10\%,3) + 2950000 (P/A,10\% 5) (P/F,10\%,3)$$

$$= 6838976 + 8401680 = \text{Rs. } 15240655$$

Similarly,

For option 5: (J_0, 3) (J, 4) (J, 1)

$$PW(10\%)_5 = 2750000 (P/A,10\%,3) + 2915000 (P/A,10\% 4) (P/F,10\%,3) + 3750000 (P/A, 10\%,1) (P/F,10\%,7) = \text{Rs. } 15530745$$

For option 6: (J_0, 4) (J, 4)

$$PW(10\%)_6 = 2980000 (P/A,10\%,3) + 2915000 (P/A,10\%4) (P/F,10\%,3) + 3750000 (P/A, 10\%,1) (P/F,10\%,7) = \text{Rs. } 15757395$$

So, by comparing PW (10%) of different strategies, the minimum PW is derived by option 3. Hence best replacement strategy would be (J_0, 2) (J, 4) (J, 2) having PW (10%) of AEC = Rs. 15074220.

Old Question Solution

1. **What is the economic service life of an asset? Find the economics service life of a new electric lift truck which costs $ 20000 have a operating cost of $1000 in the first year and salvage value of $12000 at the end of the first year. For the remaining years, operating costs increases each year by 10% over the previous years' operating costs. Similarly the salvage value declines each year by 20% from the previous year's salvage value. The lift truck has a maximum life of 7 years. An over out costing of $3000 and $5000 will be required during the fifth and seventh year of service respectively. The firm's required rate of return is 15% per year.** (2069 Bhadra)

Solution:

For theory part see at [2071 Bhadra]

Now,

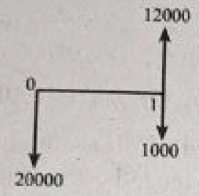
Year	Annual operating cost	Salvage
1	$ 1000	$ 12000
2	$ 1000 × (1.1) = 1100	12000 × 0.8 = 9600
3	$ 1100 × (1.1) = 1210	9600 × 0.8 = 7680
4	1331	6144
5	1664.10 (+ 3000)	4915.20
6	1610.51	3932.16
7	1771.56 (+ 3000)	3145.728

Now,

Calculating annual equivalent cost (AEC) = AEC (15%)

= CR (15%) + OC (15%)

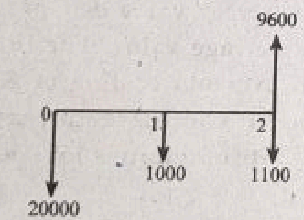
For year N = 1



$$AEC(15\%)_1 = 20000(A/P,15\%,1) - 12000(A/F,15\%,1) + 1000(A/F,15\%,1)$$

$$= 20000 \times (1.15) \times \frac{0.15}{1.15^1-1} - 12000 \times \frac{0.15}{1.15^1-1} + 1000 \times \frac{0.15}{1.15^1-1} = 23000 - 12000 + 1000 = \$\,12000$$

For year N = 2



$$AEC(15\%)_2 = (20000 \times 1.15^2 + 1000 \times 1.15 + 1100 - 9600)(A/F, 15\%,2)$$

(Note: Here first convert all cash flow to future value & convert FW to AEC)

$$= 19100(A/F, 15\%,2) = 19100 \times \frac{0.15}{1.15^2-1} = \$\,8883.72$$

Similarly,

$$AEC(15\%)_3 = (20000 \times 1.15^3 + 1000 \times 1.15^2 + 1100 \times 1.15 + 1210 - 7680)(A/F, 15\%,3)$$

$$= 26535 \times \frac{0.15}{1.15^3-1} = \$\,7641.47$$

For year N = 4

$$AEC(15\%)_3 = (20000 \times 1.15^4 + 1000 \times 1.15^3 + 1100 \times 1.15^2 + 1210 \times 1.15 + 1331 - 6144)(A/F, 15\%,4)$$

$$= 34061.85 \times \frac{0.15}{1.15^4-1} = \$\,6821.42$$

For year N = 5

[You can direct write AEC (15%) by solving on calculator similarly as above, No need to write full expression]

At yearn = 5 over head cost of $ 3000 is required so,

$$AEC(15\%) = (20000 \times 1.15^5 + 1000 \times 1.15^4 + 1100 \times 1.15^3 + 1210 \times 1.15^2 + 1331 \times 1.15 + 1664.10 + 3000 - 4915.20) \times (A/F, 15\%,5)$$

$$= 46328 \times \frac{0.15}{1.15^5-1} = \$\,6871.30$$

For year N = 6, $AEC(15\%)_6 = \$\,6465.939$

For year N = 7, $AEC(15\%)_7 = \$\,5808.31$

So, From above calculation AEC is decreases from year N = 1 to N = 7

So, economic service life = 7 years.

OR

A firm has a contract to provide printing service to IOE for next 8 years. It can provide the service using its old printing machine (the current defender) or the newly bought machine (the challenger). After the contract work neither the old machine nor the new machine will be retained. Considering the annual equivalent costs of the old machine and new machine as follows, what are their economic service life? And what is the best replacement strategy?

Number of years (N)	Annual equivalent cost (Rs) at MARR = 10%	
	Old machine	New machine
1	5,15,000	7,50,000
2	5,10,000	6,15,000
3	5,50,000	5,86,000
4	5,96,000	5,83,000
5	6,44,000	5,90,000

(2069 Bhadra)

Solution:

From the analysis of given data of AEC for old machine & new machine the value of AEC decline from N = 1 to N = 2 & then increasing for old machine, so economic service life = 2 years

And AEC decline from N = 1 to N = 4 from new machine, so economic service life = 4 years.

Replacement strategy: ($J_0 \rightarrow$ old machine, $J \rightarrow$ new machine)

Option 1: (J_0,0) (J,4) (J,4)

$$PW(10\%)_1 = 583000\,(P/A,10\%,8)$$

$$= 583000 \times \left(\frac{1.10^8 - 1}{0.1}\right) \times \frac{1}{1.1^8} \text{ [First change A to F \& then to P]}$$

$$= \text{Rs. } 3110262$$

Option 2: (J_0, 1) (J, 4) (J, 3)

$$PW(10\%)_2 = 515000\,(P/A,10\%,1) + 583000\,(P/A,10\%,4)\,(P/F,10\%,1) + 586000\,(P/A,10\%,3)\,(P/F,10\%,5)$$

$$= 515000 \times 0.91 + 583000 \times 3.17 \times 0.91 + 586000 \times 2.49 \times 0.621 = \text{Rs. } 3053102.50$$

Option 3: (J_0, 2) (J, 4) (J, 2)

$$PW(10\%)_2 = 510000\,(P/A,10\%,2) + 583000\,(P/A,10\%,4)\,(P/F,10\%,2) + 615000\,(P/A,10\%,2)\,(P/F,10\%,6)$$

$$= 510000 \times 1.735 + 583000 \times 3.17 \times 0.826 + 615000 \times 1.73 \times 0.56 = \text{Rs. } 3014844$$

Option 4: (J_0, 3) (J, 5)

$$PW(10\%)_4 = 550000\,(P/A,10\%,3) + 590000\,(P/A,10\%,5)\,(P/F,10\%,3)$$

$$= 550000 \times 2.48 + 590000 \times 3.79 \times 0.75 = \text{Rs. } 3048131$$

Option 5: (J_0, 3) (J, 4 (J, 1)

$$PW(10\%)_5 = 55000\,(P/A,10\%,3) + 583000\,(P/A,10\%,4)\,(P/F,10\%,3) + 750000\,(P/A,10\%,1)\,(P/F,10\%,7)$$

$$= 550000 \times 2.48 + 583000 \times 3.169 \times 0.68 = \text{Rs. } 3106149$$

Option 6: (J_0,4) (J,4)

$$PW(10\%)_6 = 596000\,(P/A,10\%,4) + 583000\,(P/A,10\%,4)\,(P/F,10\%,4)$$

$$= 596000 \times 3.169 + 583000 \times 3.169 \times 0.68 = \text{Rs. } 3151479$$

When we compare PW (10%) of different strategies option, the minimum PW (10%) of Annual cost is obtained by option-3

So the best replacement strategy would be (J_0, 2) (J, 4) (J, 2)

with PW (10%) = Rs. 3014844

2. **The new machine costs 10000 operating cost 2200 in first year, then increases by 20% per year. Market value is 6000 after one year and will decline by 115% each year N = 5 years. If required, old machine can work another 3 years. Market value now is 5000 and will decline by 25% each year. Immediate over hauling to restore to operable condition costs 1200. Operating costs 2000 in the first years & increases by 1500 per year thereafter. MARR = 15%.**

   i) Find the economic service life of this machine (new)

   ii) AEC of defender is as following:

N	1	2	3	4
AEC	5380	5203	5468	5845

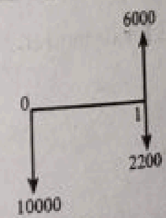
When the old machine should be replaced with the new machine.

(2070 Magh)

Solution: (i) for a new machine

No. of year	Operating cost	Salvage
1	2200	6000
2	2200 × 1.2 = 2640	6000 × 0.85 = 5100
3	2640 × 1.2 = 3168	5100 × 0.85 = 4335
4	3802	3684.75
5	4562	3132

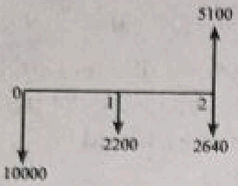
For year N = 1, AEC (15%) = CR (15%) + OC (15%)



$$AEC(15\%)_1 = [10000 \times 1.15 - 6000 + 2200]\,(A/F,15\%,1)$$

$$= 7700 \times 1 = 7700$$

For year N = 2



[Note: First convert all cash flow to FW & then into AW for AEC]

$AEC(15\%)_2 = [10000 \times 1.15^2 + 2200 \times 1.15 + 2640 - 5100]$ $(A/F, 15\%, 2)$

$= 13295 \times \frac{0.15}{1.15^2 - 1} = \text{Rs. } 6183.72$

Similarly for, year N = 3,

$AEC(15\%)_3 = (10000 \times 1.15^3 + 2200 \times 1.15^2 + 2640 \times 1.15 + 3168 - 4335)$ $(A/F, 15\%, 3)$

Also, (Just by using calculator)

For year N = 4, $AEC(15\%)_4$ = Rs. 5625.48

For year N = 5, $AEC(15\%)_5$ = Rs. 5631

So, by analyzing above calculated AEC (15%), its value decreases from year N = 1 to N = 4,

For N= 1, AEC = 7700

N= 2 , AEC = 6183.72

N= 3, AEC = 5756.09

N= 4 , AEC = 5625.48

N= 5 , AEC = 5631

So, so, economic service life = 4 years (i.e. ESL = 4 years.)

(ii) From given value of AEC for defender.

N	AEC
1	5380
2	5203
3	5468
4	5845

Here, AEC decreases from year N = 1 to N = 2 & then it increases.

So, economic service life = 2 years

Here, $AEC_{Defender} < AEC_{Challenger}$

So defender should be used for 2 years.

Now, to know the time to replace the defender we should perform marginal analysis.

If the cost of operating defender for 1 more year is more than AEC of challenger then is should be replaced.

Here, AEC of defender for economic service life (2 years) is greater than the AEC of challenges for economic service life of (5 years) so, there is no need to use defender. It can be replaced at first year.

3. **Explain about Reasons for replacement of asset. The annual equivalent of the defender and challenger are given in the table below. What is the best replacement strategy? Use MARR = 12%. The planning horizon of the project is 8 years.**

EOY (n)	1	2	3	4	5	6
$(AEC)_D$	5300	5250	5400	5750	6200	6550
$(AEC)_C$	7700	6150	5700	5600	5675	5800

(2070 Bhadra)

Solution:

For theory part See at (5.1.1)

As per above [2069 Bhadra] we can make 6 numbers of strategy but the economic strategy lies within the economic service life of defender and challenger we it has 3 strategy as.

$J_0 \rightarrow$ Defender

$J \rightarrow$ Challenger

Option 1: $(J_0, 2)$ $(J, 6)$

Option 2: $(J, 3)$ $(J, 5)$

Option 3: $(J, 4)$ $(J, 4)$

Option 1: $(J_0, 2)$ $(J, 6)$

$PW(12\%)_1 = 5250(P/A, 12\%, 2) + 5800(P/A, 12\%, 6)(P/F, 12\%, 2)$

$= 5250 \times 1.69 + 5800 \times 4.11 \times 0.79 = \text{Rs. } 27704.52$

Option 2: $(J_0, 3)$ $(J, 5)$

PW (12%)$_2$ = 5400 (P/A,12%,3) + 5675 (P/A,12%,5) (P/F, 12%,3)
= 5400 × 2.40 + 5675 × 3.6 × 0.712 = Rs. 27506.16

Option 3: (J_0, 4) (J,4)

PW (12%)$_3$ = 5750 (P/A,12%,4) + 5600 (P/A,12%,4) (P/F, 12%,4)
= 5750 × 3.037 + 5600 × 3.037 × 0.63 = Rs. 28177.286

So, by comparison of PW (12%) of annual cost for above strategy, option 3 has minimum value of PW = Rs. 27506.16

Therefore the best replacement strategy is (J_0, 3) (J, 5)

4. **Annual equivalent cost of defender and challenger are given below.**

n	Defender	Challenger
1	5380	7700
2	5203	6184
3	5469	5756
4	5844	5625
5	6258	5631
6	6682	5721

Either the defender or challenger is required for next 8 years. After the work, neither the defender nor the challenger will be retained. What is the best replacement strategy? (2071 Magh)

Solution:

$J_0 \rightarrow$ Defender $\qquad$ $J \rightarrow$ Challenger

Strategy are: Economic service life for Defender → 2 yrs
Economic service life for challenger → 4 yrs

Option 1: (J_0, 2) (J, 6)

Option 2: (J_0, 3) (J, 5)

Option 3: (J_0, 4) (J, 4)

[No need to check for other options such as (J_0, 0) (J, 4) (J, 4), (J_0, 3) (J, 4) (J, 1) etc.]

Calculating PW (i %) of Annual cost for combination as;

Option 1: (J_0, 2) (J, 6)

Consider i = 12%

PW (12%)$_1$ = 5203 (P/A,12%,2) + 5721 (P/A,12%,6) (P/F,12%,2)
= 5203 × 1.69 + 5721 × 3.277 = Rs. 27540.78

Option 2: (J_0, 3) (J, 5)

PW (12%)$_2$ = 5469 (P/A,12%,3) + 5631 (P/A,12%,5) (P/F,12%,3)
= 5469 × 2.4 + 5631 × 3.6 × 0.712= Rs. 27558.98

Option 3: (J_0,4) (J,4)

PW (12%)$_3$ = 5844 (P/A,12%,4) + 5625 (P/A,12%,4) (P/F,12%,4)
= 5844 × 3.037 + 5625 × 3.037 × 0.636 = Rs. 28613.09

So, By comparison of PW (12%), the best strategy is option 1 having least PW = Rs. 27540.78

And the replacement strategy is (J_0, 2) (J, 6)

5. **Explain about sink cost, economic life and reasons for replacement of an asset. The AEC for defender and challenger are given below. What is the best replacement strategy? Take MARR = 10% & the planning horizon of 8 years.** (2071 Bhadra)

Solution:

Sink cost: A sink cost is money that has already been spent and cannot be recovered. Sink costs are also called retrospective costs.

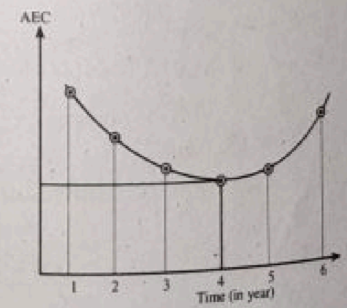
Companies in every industry have to spend money to make money. A company budget may allow for investing money in employee salaries, inventory, office space or any other cost of doing business. Once the company's money is spent, that money is considered a sink cost. Regardless of what money is spent on, sink costs are already spent & permanently lost. It cannot be refunded. For example, once rent is paid, that money is no longer recoverable which is sink.

Economic service life: The economic service life of an asset is defined as the period of useful life that minimize the annual equivalent cost (AEC), owning and operating the asset. To get exact economic service life, we need to find the value of N (years) that minimizes AEC as expressed in

AEC (i %) = CR (i %) + OC (i %)

Where CR = Capital recovery cost, OC = Operating cost

Example:



Here, life period = 6 yrs
But economic service life = 4 yrs (upto which AEC is on declining & after which AEC is increasing)
Reasons for replacement: See at [5.1.1]
For Numerical part: Refer to [2070 Bhadra]

6. An existing machine has market value of Rs. 10000 and decreases by Rs. 2000 per year. Its operating cost is Rs. 2500 in year 1 and increases by 20% each year for 4 years. New machine costs Rs. 20000 now and its market value will decrease by Rs. 20% per year from 4 years. Operating cost is Rs. 1500 in first year and increases by 30% each year. Calculate equivalent uniform annual cost of both existing and new machines. MAPR = 15%. Formulate the best replacement strategy if we need the machine for four years only.

(2072 Ashwin)

Solution:

	For Defender		For challenger	
Year	Operation cost	Salvage	Operation cost	Salvage
0	–	10000	–	20000
1	2500	8000	1500	$20000 \times 0.8 = 16000$
2	3000	6000	1950	12800
3	3600	4000	2535	10240
4	4320	2000	3295.5	8192

For Defender:

For N = 1 $AEC(15\%)_1 = CR(15\%) + OC(15\%)$

$= [10000\,(F/P,15\%,1) - 8000 + 2500] \times (A/F,15\%,1)$

$= (10000 \times 1.15 - 8000 + 2500) \times \frac{0.15}{1.15^1 - 1} =$ Rs. 6000

For N = 2 $AEC(15\%)_2 = (10000 \times 1.15^2 - 6000 + 2500 \times 1.15 + 3000)\,(A/F, 15\%,2)$

$= 13100 \times 0.465 =$ Rs. 6093.02

For N = 3 $AEC(15\%)_3 = (10000 \times 1.15^3 - 4000 + 2500 \times 1.15^2 + 3000 \times 1.15 + 3600)\,(A/F, 15\%,3)$

$= 21565 \times 0.2879 =$ Rs. 6210.22

For N = 4 $AEC(15\%)_3$ = (Directly from calculator) = Rs. 6352.36

For challenger:

For N = 1, $AEC(15\%)_1 = (20000 \times 1.15 - 16000 + 1500)\,(A/F,15\%,1)$

= Rs. 8500

For N = 2, $AEC(15\%)_2 = (20000 \times 1.15^2 - 12800 + 1500 \times 1.15 + 1950) \times (A/F,15\%,2)$

= Rs. 8058.14

For N = 3, $AEC(15\%)_3 = (20000 \times 1.15^3 - 10240 + 1500 \times 1.15^2 + 1950 \times 1.15 + 2535) \times (A/F, 15\%,3) =$ Rs. 7757.74

For N = 4, $AEC(15\%)_4 = (20000 \times 1.15^4 - 8192 + 1500 \times 1.15^3 + 1950 \times 1.15^2 + 2535 \times 1.15 + 3295.5) \times (A/F, 15\%,4)$

= Rs. 7581.858

So from above calculation,
Economic service life of defender is 1 year
Economic service life of challenge is 4 years
But AEC of challenger > AEC of defender for Economic service life.
Also, maximum AEC of defender < AEC of challenges for economic service life.
Therefore No need to replace the existing machine.

7. What are procedure for replacement analysis when planning horizon is infinite?
(2073 Bhadra)

Solution:
Refer [5.3.2]

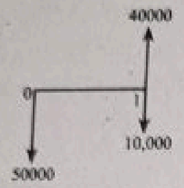
8. Find economic service life from the following information.
Initial cost = Rs. 50,000
Operation cost = Rs. 10,000 for the 1st year and increases by 15% Thereafter,
Salvage value = Decline each successive year by 20% over previous year.
Useful life = 8 years
MARR = 15%
(2073 Bhadra)

Solution:

Year	Annual Operating Cost	Salvage
1	10000	$50000 \times 0.8 = 40000$
2	$10000 \times 1.15 = 11500$	$40000 \times 0.8 = 32000$
3	$11500 \times 1.15 = 13225$	$32000 \times 0.8 = 25600$
4	$13225 \times 1.15 = 15208.75$	20480
5	17490	16384
6	20113.57	13107.2
7	23130.60	10485.76
8	26600.19	8388.60

Now, calculating annual equivalent cost (AEC) = AEC (15%)
= CR (15%) + OC (15%)

For year N = 1

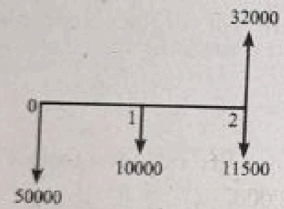


$$AEC\,(15\%)_1 = 50000\,(A/P, 15\%, 1) - 40000\,(A/F, 15\%, 1) + 10000\,(A/F, 5\%, 1)$$

$$= 50000 \times 1.15 \times \frac{0.15}{1.15^1 - 1} - (40000 + 10000) \times \frac{0.15}{1.15^1 - 1}$$

= Rs. 27500

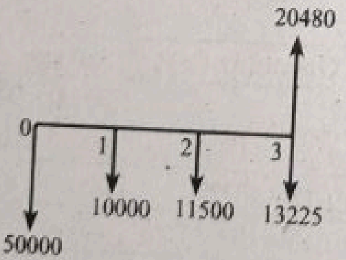
For year N = 2



$$AEC\,(15\%)_2 = (50000 \times 1.15^2 + 10000 \times 1.15 + 11500 - 32000)\,(A/F, 15\%, 2)$$

$$= 57125 \times \frac{0.15}{1.15^2 - 1} = \text{Rs. } 26569.76$$

Similarly, for N = 3



$$AEC\,(15\%)_3 = (50000 \times 1.15^3 + 10000 \times 1.15^2 + 11500 \times 1.15 + 13225 - 20480)\,(A/F, 15\%, 3)$$

= Rs. 27426.56

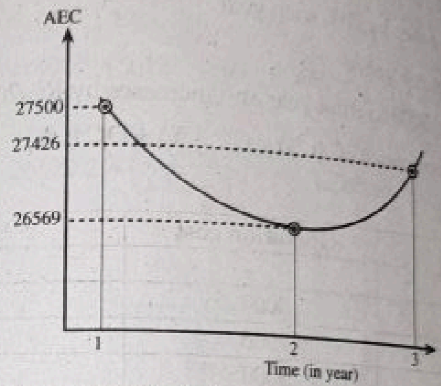
Here, For N = 1, AEC $(15\%)_1$ = Rs. 27500

For N = 2, AEC $(15\%)_2$ = Rs. 26569.76

For N = 3, AEC $(15\%)_3$ = Rs. 27426.56

So from above calculation AEC decreases from year 1 to 2 & then again increases from 2 to 3.

In graph



Therefore economic service life is 2 years.

9. **Define sunk cost and economic life of an assets. What are the reasons for replacement of asset? The Annual Equivalent cost of defender and challenger and given in table below. What is the best replacement strategies? Use MARR = 12% and planning horizon of the project is 8 years.**

EOY (n)	1	2	3	4	5	6
(ACE) (Rs)	5300	5100	5400	5600	6000	6500
(ACE) (Rs)	2500	6000	5600	5500	5650	5800

(2073 Magh)

Solution:

First part refer 5.1.1

[Note: Past participle of sink is sunk)

Second Part: Refer 2070 Bhadra

10. **Define defender and challenger and explain economic service life. Company X is going to purchase a router having initial cost Rs. 18000 having salvage value of Rs. 12000 at the end of first year and decreases by 20% each year then after for remaining useful life. Annual operation and maintenance cost is Rs. 5000 in first year and increases by Rs. 2000 each year. Its useful life is 6 years. Calculate economic service life of the router. (2074 Bhadra)**

Solution:

Refer chapter content for definition part.

Given, I = 18,000, Assuming MARR = 12% per year

At the end of first year salvage value = 12000

And decreases by 20% each year

Useful life = 6 years

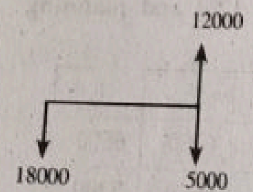
Operation cost = 5000 for first year and increases by Rs. 2000 after then

Annual equivalent cost AEC (i %) = CR (i %) + OC (i %)

Now,

N	Operation cost	Salvage
0	–	18000
1	5000	12000
2	7000	9600
3	9000	7680
4	11000	6144
5	13000	4915.2
6	15000	3932.16

For year N = 1



AEC (12%) = CR (12%) + OC (12%)

= [18000 (F/P, 12%, 1) – 12000 + 9000] × (A/F, 12%, 1)

$= (18000 \times 1.12 - 12000 + 9000) \times \frac{0.12}{1.12^1 - 1}$

= Rs. 17160

Note: We can also calculate CR and OC separately as,

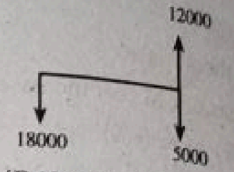
CR (12%) = 18000 (A/P, 12%, 1) – 12000 (A/F, 12%, 1)

= 12160

OC (12%) = 5000

Then AEC (12%) = 12160 + 5000 = 17160

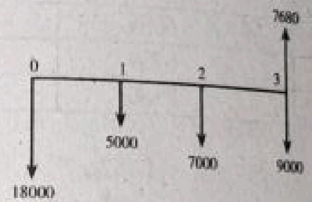
For N = 2



$AEC(12\%)_2$ = [18000 (F/P,12%,2) – 9600 + 5000 (F/P, 12%, 1) + 7000] × (A/F, 12%, 2)

$= (18000 \times 1.12^2 - 9600 + 5000 \times 1.12 + 7000) \times (A/F, 12\%, 2)$

$= 25579.2 \times \left(\frac{0.12}{1.12^2 - 1}\right)$ = Rs. 12065.66



Similarly, For N = 3

$AEC(12\%)_3 = 12067.53$

For N = 4, $AEC(12\%)_4 = 12358.38$

For N = 5, $AEC(12\%)_5 = 12768.89$

For N = 6, $AEC(12\%)_6 = 13084.72$

[Note: Value is directly calculated by using calculator]

So Here AEC is decreases from N = 1 to 2, then it started to increase.

That means economic service life of router is 2 years.

11. **What do you mean by replacement analysis and economic service life? What are the procedure when planning horizon is infinite and finite? Calculate AECs from the following information and determine service life.** (2075 Baisakh)

I = 18,000

N = 8 years

O & M = 3000 for the 1st year and increases by 15% thereafter

S = Decline by 20% each successive year over than previous price

MARR = 12% per year

Solution:

Refer chapter content for theory part.

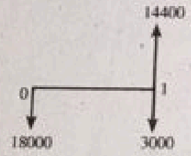
Given: MARR=12%per year,O& M cost increases by 15%,S decline by 20%

$$AEC\ (i\ \%) = CR\ (i\ \%) + OC\ (i\ \%)$$

Now,

N	Operation cost	Salvage
0	0	18000
1	3000	14400
2	3450	11520
3	3967.5	9216
4	4562.62	7372.8
5	5247	5898.24
6	6034	4718.59
7	6939	3774.82
8	7980	3020

For year N = 1

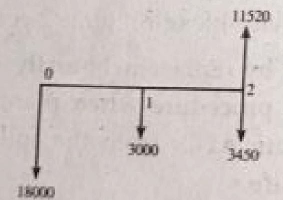


$$AEC\ (12\%)_1 = CR\ (12\%) + OC\ (12\%)$$

$$= [18000\ (F/P, 12\%, 1) - 14400 + 3000] \times (A/F, 12\%, 1)$$

$$= (18000 \times 1.12 - 14400 + 3000) \times \frac{0.12}{1.12^1 - 1} = \text{Rs. } 8760$$

For year N = 2

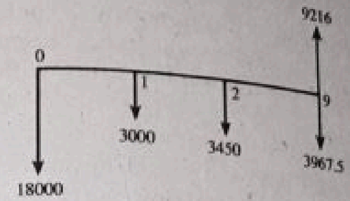


$$AEC\ (12\%)_2 = [18000\ (F/P, 12\%, 2) - 11520 + 3000\ (F/P, 12\%,1) + 3450] \times (A/F, 12\%, 2)$$

$$= (18000 \times 1.12^2 - 11520 + 3000 \times 1.12 + 3450) \times \frac{0.12}{1.12^2 - 1}$$

$$= 17869.2 \times \frac{0.12}{1.12^2 - 1} = \text{Rs. } 8429$$

For N = 3



$$AEC\ (12\%)_3 = [18000\ (F/P, 12\%, 3) - 9216 + 3000\ (F/P, 12\%,2) + 3450\ (F/P,12\%,1) + 3967.5] \times (A/F, 12\%, 3)$$

$$= (18000 \times 1.12^3 - 9216 + 3000 \times 1.12^2 + 3450 \times 1.12 + 3967.5) \times \frac{0.12}{1.12^3 - 1} = \text{Rs. } 8199$$

Similarly,

For N = 4, $AEC\ (12\%)_4$ = Rs. 8055

For N = 5, $AEC\ (12\%)_5$ = Rs. 7985

For N = 6, $AEC\ (12\%)_6$ = Rs. 7977

For N = 7, $AEC\ (12\%)_7$ = Rs. 8024

For N = 8, $AEC\ (12\%)_8$ = Rs. 8118

Here, AEC (12%) is decreases from N = 1 to 6 and started to increase after then. That is the economic service life = 6 years

But useful life = 8 years (as given)

12. **What is replacement analysis? What factors should be considered in replacement analysis? Explain the cash flow approach and opportunity cost approach.** (2076 Bhadra)

Solution: Refer introduction part and 5.1.2.

13. **A new machine costs $20,000. Future market values are expected to decrease by $ 2000 each year over the previous year's value. Useful life of the machine is six years. Operating costs are estimated at $ 3000 during the first year and expected to increase 15% per year thereafter. MARR 12%. Determine the economic service life of the machine.** (2076 Bhadra)

Solution:

Refer the solution of (2074 Bhadra)

Extra Questions solution

1. **Advanced Electrical Insulator Company is considering replacing a broken inspection machine, which has been used to test the mechanical strength of electrical insulators, with a newer and more efficient one.**

- If repaired, the old machine can be used for another five years, although the firm does not expect to realize any salvage value from scrapping it at that time. However, the firm can sell it now to another firm in the industry for $5,000. If the machine is kept, it will require an immediate $1,200 overhaul to restore it to operable condition. The overhaul will neither extend the service life originally estimated nor increase the value of the inspection machine. The operating costs are estimated at $2,000 during the first year, and these are expected to increase by $1,500 per year thereafter. Future market values are expected to decline by $1,000 per year.
- The new machine costs $10,000 and will have operating costs of $2,000 in the first year, increasing by $800 per year thereafter. The expected salvage value is $6,000 after one year and will decline 15% each year. The company requires a rate of return of 15%. Find the economic life for each option, *and* determine when the defender should be replaced.

Solution:

1. Economic service life

Defender: If the company retains the inspection machine, it is in effect deciding to overhaul the machine and invest the machine's current market value in that alternative. The opportunity cost of the machine is $ 5,000. Because an overhaul costing $ 1,200 is also needed to make the machine operational, the total initial investment in the machine is $ 5,000 + $1,200 = $ 6,200. Other data for the defender are summarized as follows:

n	Overhaul	Forecasted Operating Cost	Market Value if Disposed of
0	$1,200		$5,000
1	0	$2,000	$4,000
2	0	$3,500	$3,000
3	0	$5,000	$2,000
4	0	$6,500	$1,000
5	0	$8,000	0

We can calculate the annual equivalent costs if the defender is to be kept for one year, two years, three years, and so forth. For example, the cash

flow diagram for N = 4 years is shown in figure below. The Annual equivalent costs for four years are as follows:

N = 4 years: ACE (15%) = $ 6,200 (A/P, 15%, 4) + $ 2,000 + $ 1,500 (A/G, 15%, 4) – $ 1,000 (A/F, 15%, 4) = $ 5,961

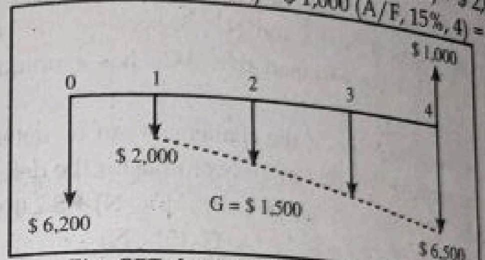


Fig: CFD for defender when N=4 Years

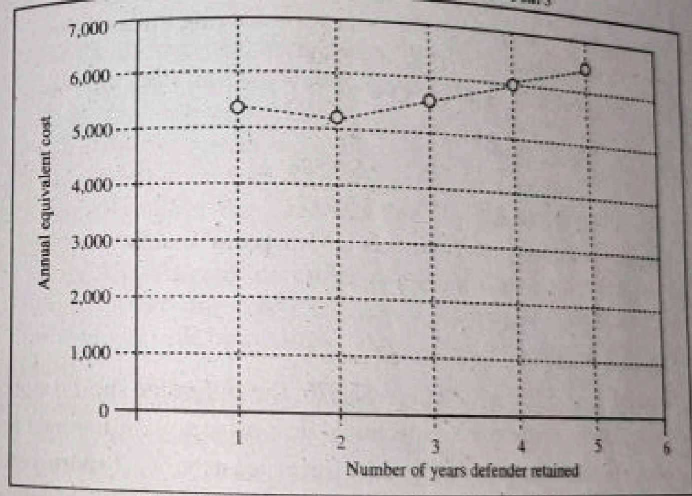


Fig: AEC as the function of the life of a defender

The other AE cost figures can be calculated with the following equation:

AEC (15%)$_N$ = $ 6,200 (A/P, 15%, N) + $ 2,000 + $ 1,500 (A/G, 15%, N) – $ 1,000 (5 – N) (A/F, 15%, N) for N = 1, 2, 3, 4, 5;

N = 1 : ACE (15%) = $ 5,130,
N = 2 : ACE (15%) = $ 5,116,
N = 3 : ACE (15%) = $ 5,500,
N = 4 : ACE (15%) = $ 5,961,
N = 5 : ACE (15%) = $ 6,434,

When N = 2 years, we get the lowest AEC value. Thus, the defender's economic life is two years. Using the notation we defined in the procedure, we have

$N_D^* = 2$ years,

$AEC_D = \$5,116$

The AEC values as a function of are plotted in figure 14.8. Actually, after computing ACE for N = 1, 2, and 3, we can stop right there. There is no need to compute ACE for N = 4 and N = 5, because AEC is increasing when N > 2 and we have assumed that ACE has a unique minimum point.

Challenger. The economic life of the challenger can be determined with the same procedure we used in this example for the defender.

$$AEC(15\%)_N = \$10,000 (A/P, 15\%, N) + \$2,000 + \$800 (A/G, 15\%, N) - \$6,000 (1 - 15\%)^{N-1} (A/F, 15\%, N)$$

The results of "plugging in" the values and solving are as follows:

N = 1 year: AEC (15%) = $ 7,500

N = 2 years: AEC (15%) = $ 6,151

N = 3 years: AEC (15%) = $ 5,857

N = 4 years: AEC (15%) = $ 5,826

N = 5 year: AEC (15%) = $ 5,897

The economic life of the challenger is four years: that is

$N_C^* = 4$ years

Thus, $AEC_C^* = \$5,826$

2. **Should the defender be replaced now?**

Since $AEC_D^* = \$5,116 < AEC_C = \$5,826$, the defender should not be replaced now. If there are no technological advances in the next few years, the defender should be used for at least $N_D^* = 2$ more years. However, it is not necessarily best to replace the defender right at the high point of its economic life.

3. **When should the defender be replaced?**

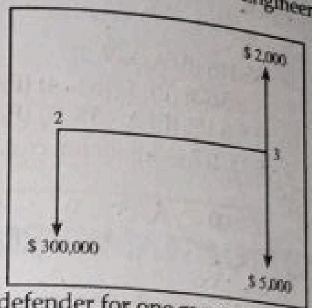
If we need to find the answer to this question today, we have to calculate the cost of keeping and using the defender for the third year from today. That is, what is the cost of not selling the defender at the end of year 2, using it for the third year, and replacing it at the end of year 3? The following cash flows are related to this question:

(a) Opportunity cost at the end of year: equal to the market value or & 3,000.

(b) Operating cost for the third year: $ 5,000.

(c) Salvage value of the defender at the end of year 3:$ 2, 000

The following diagram represents these cash flows:



The cost of using the defender for one more year from the end of its economic life

$$\$3,000 \times 1.15 + \$5,000 - \$2,000 = \$6,450.$$

Now compare this cost with the $AEC_C^* = \$5,826$ of the challenger. It is greater than AEC_C^*. Thus, it is more expensive to keep the defender for the third year than to replace it with the challenger. Accordingly, we conclude that we should replace the defender at the end of year 2. If this one-year cost is still smaller than AEC_C^*, we need to calculate the cost of using the defender for the fourth year and then compare that cost with the AEC_C^* of the challenger.

2. **Consider again the defender and the challenger in above Q.N. 1 Suppose that the firm has a contract to perform a given service, using the current defender or the challenger for the next eight years. After the contract work, neither the defender nor the challenger will be retained. What is the best replacement strategy?**

Solution:

Recall again the annual equivalent costs for the defender and challenger under the assumed holding periods (a boxed number denotes the minimum AEC value at $N_D^* = 2$ and $N_C^* = 4$, respectively):

Annual Equivalent Cost ($)

n	Defender	Challenger
1	5,130	7,500
2	[5,116]	6,151
3	5,500	5,857
4	5,961	[5,826]
5	6,434	5,897

Option 1: $(j_0, 0), (j, 4), (j, 4)$

$$PW(15\%)_1 = \$5,826 (P/A, 15\%, 8) = \$26,143$$

Option 2: $(j_0, 1), (j, 4), (j, 3)$

$$PW(15\%)_2 = \$5,130 (P/E, 15\%, 1) + \$5,826 (P/A, 15\%, 4)(P/F, 15\%, 1) + \$5,857 (P/A, 15\%, 3)(P/F, 15\%, 5) = \$26,143$$

Option 3: (j_0, 2), (j, 4), (j, 2)

PW (15%)$_3$ = \$5,116 (P/A, 15%,2)
+ \$ 5,826 (P/A, 15%, 4) (P/F, 15%, 2)
+ \$ 6,151 (P/A, 15%, 2) (P/F, 15%,6)
= \$ 25,217 ← minimum cost.

Option 4: (j_0, 3), (j, 5)

PW (15%)$_4$ = \$5,500 (P/A, 15%, 3)
+ \$ 5,897 (P/A, 15%, 5) (P/F, 15%, 3)
= \$ 25,555

Option 5: (j_0, 3), (j, 4), (j, 1)

PW (15%)$_5$ = \$5,500 (P/A, 15%, 3)
+ \$ 5,826 (P/A, 15%, 5) (P/F, 15%, 3)
+ \$ 7,500 (P/F, 15%, 8) = \$ 25,946

Option 6: (j_0, 4), (j, 4)

PW (15%)$_5$ = \$5,961 (P/A, 15%, 4)
+ \$ 5,826 (P/A, 15%, 4) (P/F, 15%, 4)
= \$ 26,529

An examination of the present equivalent cost of a planning horizon of eight years indicates that the least-cost solution appears to be option 3: Retain the defender for two years, purchase the challenger the keep it for four years, and purchase another challenger and keep it for two years.

6 RISK MANAGMENET

6.1 Origin / Sources of Project Risks

Risk: The chance that on investment's actual return will be different than expected:

The decision to make a major capital investment such as introducing a new product requires information about cash flow over the life of a project. The profitability estimate of an investment depends on cash flow estimations, which are generally uncertain. The factors to be estimated include the total market for the product; the market share that the firm can attain; the growth in the market; the cost of producing the product, including labor and materials; the selling price; the life of the product; the cost and life of the equipment needed; and the effective tax rates. Many of these factors are subject to substantial uncertainty. A common approach is to make single-number "best estimates" for each of the uncertain factors and then to calculate measures of profitability, such as the NPW or rate of return for the project. This approach, however, has two drawbacks:

1. No guarantee can ever ensure that the "best estimates" will match actual values.
2. No provision is made to measure the risk associated with an investment, or the project risk. In particular, managers have no way of determining either the probability that a project will lose money or the probability that it will generate large profits.

Because cash flows can be so difficult to estimate accurately, project managers frequently consider a range of possible values for cash flow elements. If a range of values for individual cash flows is possible, it follows that a range of values for the NPW of a given project is also possible. Clearly, the analyst will want to gauge the probability and reliability of individual cash flows and, consequently, the level of certainty about the overall project worth.

In summary origin of project risk can be presented as

1) **Cash Flow Estimate :** Inaccuracy of the cash flow estimates and measurements error
2) **Nature of Business :** All business are not same nature because some types of business operation are less stable then others.
3) **Rate of Interest :** It depends on current health of economy and future expectation of economic condition.
4) **Study Period :** Long study period generally increases the uncertainty of a capital investment and economic return.
5) **Unclear specification**
6) **Volatile and unpredictable future**
7) **Social risks**
8) **Variability in output**

6.2 Methods of Describing Project Risk

There are various methods for describing projects risk, listed as:

1. Sensitivity Analysis
2. Break even Analysis
3. Scenario Analysis

6.2.1 Sensitivity Analysis

Sensitivity analysis is a general non probabilistic methodology readily available, to provide information about potential impact of uncertainty in selected factor estimates. Its routine use is fundamental to developing economic information useful in the decision process. In general sensitivity means the relative magnitude of change in the measure of merit. (PW or IRR) caused by one or more changes in estimated study factor values.

It is a means of identifying the project variables which when varied, have the greatest effect on project acceptability.

Sensitivity analysis is sometimes called "what-if" analysis, because it

answers questions such as "What if incremental sales are only 1,000 units, rather than 2,000 units? Then what will the NPW be?" Sensitivity analysis begins with a base-case situation, which is developed by using the most likely values for each input. We then change the specific variable of interest by several specified percentage points above and below the most likely value, while holding other variables constant. Next, we calculate a new

NPW for each of the values we obtained. A convenient and useful way to present the results of a sensitivity analysis is to plot **sensitivity graphs**. The slopes of the lines show how sensitive the NPW is to changes in each of the inputs: The steeper the slope, the more sensitive the NPW is to a change in a particular variable. Sensitivity graphs identify the crucial variables that affect the final outcome most.

Following technique are usually included in a discussion of sensitivity analysis.

i) Breakeven analysis
ii) Sensitivity graph
iii) Combination of factors

Steps for sensitivity graph analysis:

a) Plot PW, FW, AW, IRR or BCR on y - axis
b) % error in estimate of parameter value is plotted on x - axis
c) The slope of the line shows level of sensitivity.
d) The steeper the slope, more sensitive.

Example: A Project costs Rs. 500000 and will have annual saving of Rs. 92500 per year for 10 years, salvage value is 10% of initial cost. Perform sensitivity analysis over a period of ± 40% in

i) **Initial investment**
ii) **Annual Saving**
iii) **Project Life**

Assume MARR = 10% using AW formulation

Solution:

Salvage = 10% of I = 50,000

AW = - 500000 (A/P,10 %, 10) + 92500 + 50000 (A/F, 10% 10)

$$= -500000 \times \frac{0.1 \times 1.1^{10}}{1.1^{10} - 1} + 92500 + 50,000 \times \frac{0.1}{1.1^{10} - 1}$$

$$= 14264.57$$

Writing in equation format

$$AW = -I\,(A/P, 10\%, N) + B + 50,000\,(A/F, 10\%, N)$$

$$= -I * \frac{0.1 \times 1.1^{N}}{1.1^{N} - 1} + B + 50,000 \times \frac{0.1}{1.1^{N} - 1} \quad \text{.......... (i)}$$

Where I = initial investment, B = annual saving, N = project life

Variation	10%	20%	30%	40%	-10%	-20%	-30%	-40%	Remarks
AW (I varies)	6127.3	-2009	-10147.2	-18284	22401.8	30539.1	38676.3	46813.7	B & N Constant
AW (B varies)	23514.57	32764.57	4204.57	51264.57	5014.57	-4235.47	-13485.42	-22735.4	I and N constant
AW (N varies)	18216.58	21456.5	2449.6	26414.2	9361.75	3150.19	-4932.47	-15823.23	B & I constant

Calculator Trick

For I varies

Enter equation (i) in calculator

i.e. $-x * \frac{0.1 \times 1.1^{10}}{(1.1^{10} - 1)} + 92500 + 50000 \times 0.1 / (1.1^{10} - 1)$

Press CALC, X = ?

Enter 500000 * 1.1 ⇒ 6127.3 (For 10%)

Press CALC, X = ?

Enter 500000 * 1.2 ⇒ - 2009.96 (For 20%)

And For negative sign

Enter 500000 * 0.9 ⇒ 22401.84 (For - 10%)

Enter 500000 × 0.8 ⇒ 30539.1 (For - 20%)

Similarly, For N

Enter equation (1) in this way

$$-500000 \times 0.1 \times \frac{1.1^{x}}{(1.1^{x} - 1)} + 92500 + 50000 \times \frac{0.1}{(1.1^{x} - 1)}$$

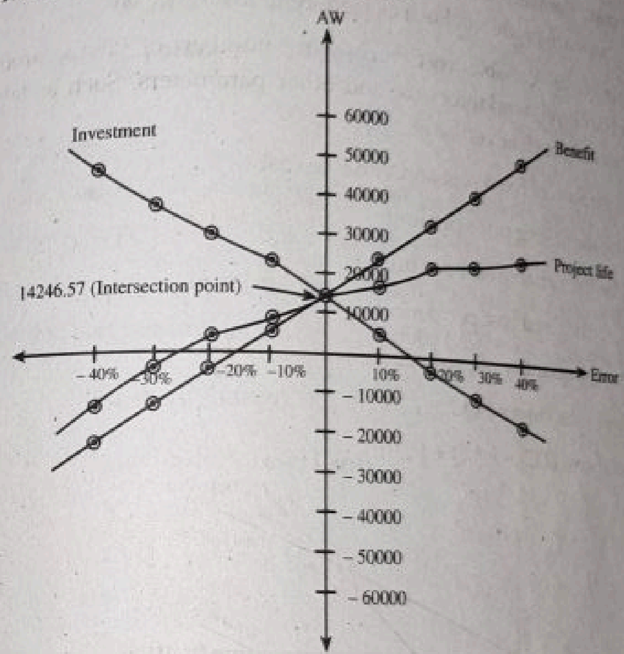
Enter CALC X ?

Enter 10* 1.1 ⇒ 18216.58 (For 10%) and Similar for other

Enter 10 × 0.9 ⇒ 9361.15 (For - 10%)

And Similar for B

And fill in the table and plot the graph



From graph benefit has a higher slope therefore, the most sensitive is benefit.

6.2.2 Breakeven Analysis

The usual breakeven problem involving two alternatives can be most easily approached mathematically by equating an equivalent worth of the two alternatives expressed as a function of the factor of interest. In short, the break - even point is the point at which revenue is exactly equal to costs.

Break even analysis is mean of identifying the value of a particular project variable that causes the project exactly to break ever.

Breakeven Volume

It is the amount of product that a firm need to produce and sell to cover total cost of production. This can be computed by

$$\text{Breakeven Volume} = \frac{\text{Total Fixed Cost}}{\text{elling Price - Variable Cost per Unit}}$$

Fixed cost: Remain constant over how many times you sell, and how many goods are produced. Such as land, rent, insurance etc.

Variable cost: Variable cost vary with number of items produce, production level, workforce size and other parameters. Such as labour, raw materials, fuel, advertise etc.

Total cost: Sum of fixed cost and variable cost

Let S be the selling price per unit

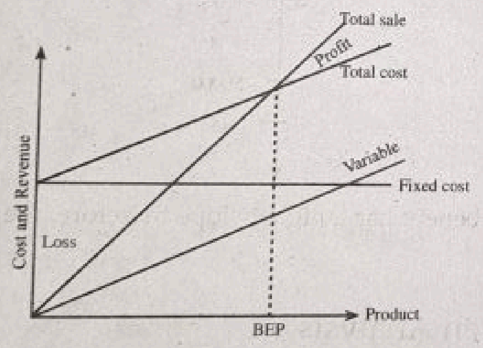
V be the variable cost per unit

FC is the fixed cost per period

Q is the quantity of production

Total sales (S) = s * Q .... (i)

Total cost (TC) = v * Q + FC (ii)



At intersection point

Total cost = total sales

v * Q + FC = s * Q

or, FC = (s – v) Q

or $Q = \frac{FC}{s - v}$

Example: From given data, determine

(a) **BEP in terms of number of unit**

(b) **What should be the output if the profit desired is Rs. 5,000?**

(c) **What is the different between revenue and variable cost per unit at breakeven?**

Fixed Cost (FC) = Rs. 4,000

Total Cost (TC) = Rs. 8,500

Total Sales (S) = Rs. 10,500

Sales Volume = 1500 units

Solution:

We have,

(a) Total Cost (TC) = Fixed Cost (FC) + Variable Cost (VC)

VC = TC – FC = 8,500 – 4,000 = 4,500

Variable cost per unit (v) = 4,500 / 1,500 = Rs. 3 per unit

Selling cost per unit (s) = 10500/1,500 = Rs. 7 per unit

QBEP = FC/ (s – v) = $\frac{4000}{7-3}$ = 1,000 units

(b) Profit = Total Sales - Total Cost

5,000 = s * Q – (FC + VC)

= s * Q – (400 + v * Q)

5000 + 4000 = (s – v) Q

Q = $\frac{9000}{7-3}$ = 2,250

Therefore break even unit for the profit to be Rs. 5,000 is 2,250

(c) At break even profit = 0, Assume FC remains constant

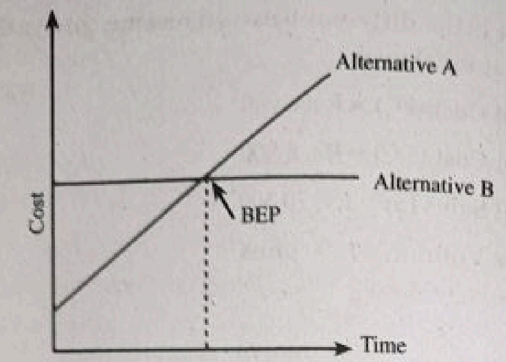
0 = s * Q – (4000 + v * Q)

4000 = (s – v) * 2000

s – v = Rs. 2 per unit

Break - Even Analysis for comparing two alternatives

When there are two investment opportunity under consideration and heavily dependent on a single and common factor obtained. This value by equating equivalent worth. That value is known as the break - even point.



Mathematically,

E_{WA} = f_1 (y) and E_{WB} = f_2 (y)

Where, E_{WA} = Equivalent worth calculation for the net cash flow of alternative A.

F_{WB} = Equivalent worth calculation for the net cash flow of alternative B

y = A common factor affecting the equivalent worth values of alternative A and alternative B

At break-even point,

E_{WA} = E_{WB}

$\Rightarrow f_1(y) = f_2(y)$

Example

Consider the following two motors each of 100 hp output capacity item.

Item	Motor A	Motor B
Purchase Cost (Rs)	1,25,000	1,60,000
Efficiency (η)	74%	92%
Life (Years)	10	10
Maintenance cost (Rs)/Year	5,000	2,500
Annual tax and insurance	1.5% of investment	1.5 of investment
MARR	15%	15%

(a) **How many hours per year would the motors have to be operated at full load for the annual costs to be equal? If electricity cost is Rs. 5/KW/hr.**

(b) **If annual operation is more than 55 hours, which motor should be selected?**

Solution:

For Motor 'A'

Calculating the annual equivalent cost

CR (Capital recovery) cost = 1,25,00 (A 1P, 15%, 10)

= Rs. 24,906.5

Maintenance cost = Rs. 5,000

Tax and insurance = 1.5% of 1,25,000 = Rs. 1,875

Operating expenses for power (electricity cost),

We known that,

$$\text{Efficiency } (\eta) = \frac{\text{Output}}{\text{Input}}$$

Let 'X' be the number of hours of operation per year

Operating expenses = Input × rate * Hours

$$= \frac{\text{Output}}{h} \times \text{Rate} \times \text{Hours}$$

$$= \frac{100 \times 0.746}{0.74} \times 5 \times X \; [1 \text{ hp} = 0.746 \text{ kw}]$$

= 504.5 X

∴ Total amount equivalent cost of motor A

= 24,906.5 + 5,000 + 1,875 + 504.05X

= 31,781.5 + 504.05X

For Motor B

Capital Recover (CR) = 1,60,000 (A/P, 15%, 10)

= Rs. 31, 888.32

Maintenance cost = Rs. 2,500

Tax and insurance = 1.5% of 1,60,000

= Rs. 2,400

Operating expenses for power,

Let 'X' be the number of hours of operation per year

$$\text{Operating expenses} = \frac{100 \times 0.746}{0.92} * 5 * X$$

$$= 405.43x$$

∴ Total annual equivalent cost of motor B (AW of B)

$$= 31{,}880.32 + 2{,}500 + 2{,}400 + 405.45X$$

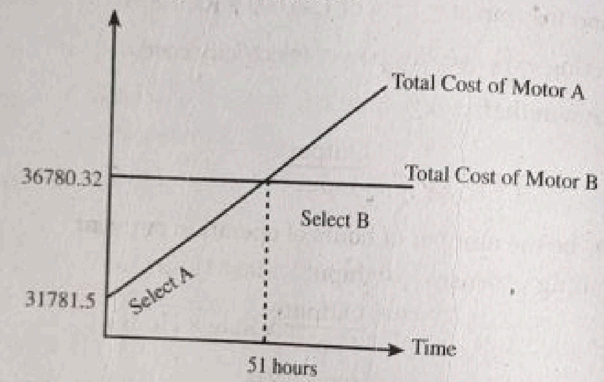
$$= 36{,}780.32 + 405.43X$$

At Breakeven point,

$$\text{AW of A} = \text{AW of B}$$

$$31{,}781.5 + 504.05\,X = 36.780.32 + 405.43X$$

$$X = 51 \text{ hours}$$



∴ If annual operation is more than 55 hours, motor B is selected.

6.2.3 Scenario Analysis

Although both sensitivity and break-even analyses are useful, they have limitations. Often, it is difficult to specify precisely the relationship between a particular variable and the NPW. The relationship is further complicated by interdependencies among the variables. Holding operating costs constant while varying unit sales may ease the analysis, but in reality, operating costs do not behave in this manner. Yet, it may complicate the analysis too much to permit movement in more than one variable at a time.

Scenario analysis is a technique that considers the sensitivity of NPW both to changes in key variables and to the range of likely values of those variables. For example, the decision maker may examine two extreme cases: a "worst-case" scenario (low unit sales, low unit price, high variable cost per unit, high fixed cost, and so on) and a "best-case" scenario. The NPWs under the worst and the best conditions are then calculated and compared with the expected, or base-case, NPW.

In summary scenario analysis is a means of comparing a "base case" or expected project measurement (such as NPW) with one or more additional scenarios, such as the best case and the worst case, to identify the extreme and most likely project outcomes.

6.3 Probability Concept of Economic Analysis

Risk analysis: It is a technique to identify and assess factors that may jeopardize the success of a project.

It assigns the probabilities of various outcomes of on investment project.

The probability distribution of a random variable allows us to make a specific probability statement, a single value that may characterize the random variable and its probability distribution is often desirable. Such a quantity is the expected value of a random variable. We also want to know something about how the values of the random variable are dispersed about the expected value. (i.e. the variance). In investment analysis this dispersion information is interpreted as the degree of project risk. The expected value indicates the weighted average of the random variable, and the variance captures the variability of the random variable.

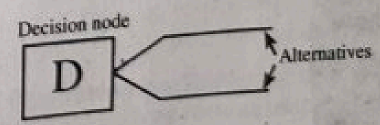
6.4 Decision Tree & Sequential Investment Decision

The assignment of probabilities to the various outcomes of an investment project is generally called **risk analysis.**

A decision tree is a device that shows a sequence of strategic decision and the expected consequences under each possible set of circumstances.

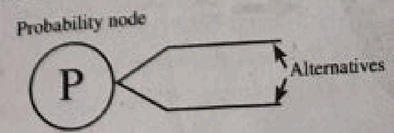
Components of Decision Tree

Decision Node:



Probability node:

A circle represents probability node with the possible outcomes and estimated probabilities on the branches.



Branch

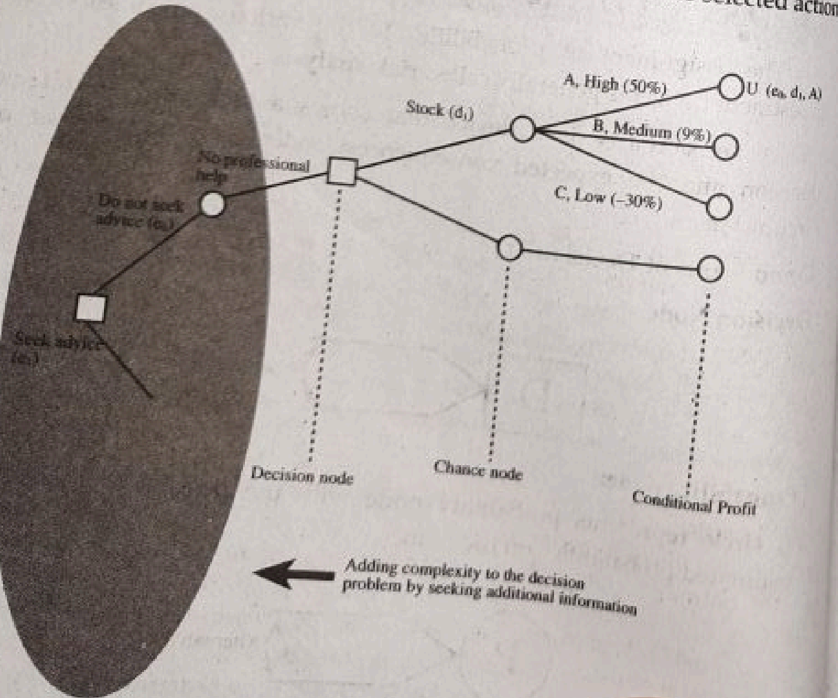
Line connecting nodes from the left to the right of the diagram.

To illustrate the basic decision-tree concept, let's consider an investor, named Bill Henderson, who wants to invest some money in the financial market. He wants to choose between a highly speculative growth stock d1 and a very safe U.S. Treasury bond d2.

Figure below illustrates this situation, with the decision point represented by a square box or decision node. The alternatives are represented as branches emanating from the node. Suppose Bill were to select some particular alternative, say, "invest in the stock."

There are three chance events that can happen, each representing a potential return on investment. These events are shown in Figure as branches emanating from a circular node. Note that the branches represent uncertain events over which Bill has no control.

However, Bill can assign a probability to each chance event and enter it beside each branch in the decision tree. At the end of each branch is the conditional monetary transaction associated with the selected action and given event.



Relevant Net After-Tax Cash Flow

Once the structure of the decision tree is determined, the next step is to find the relevant cash flow (monetary value) associated with each of the alternatives and the possible chance outcomes. As we have emphasized throughout this book, the decision has to be made on an after-tax basis. Therefore, the relevant monetary value should be on that basis. Since the costs and revenues occur at different points in time over the study period, we also need to convert the various amounts on the tree's branches to their equivalent lump-sum amounts, say, and net present value. The conditional net present value thus represents the profit associated with the decisions and events along the path from the first part of the tree to the end.

Rollback Procedure

To analyze a decision tree, we begin at the end of the tree and work backwards – the **rollback** procedure. In other words, starting at the tips of the decision tree's branches and working toward the initial node, we use the following two rules:

1. For each chance node, we calculate the expected monetary value (EMV). This is done by multiplying probabilities by conditional profits associated with branches emanating from that node and summing these conditional profits. We then place the EMV in the node to indicate that it is the expected value calculated over all branches emanating from that node.
2. For each decision node, we select the one with the highest EMV (or minimum cost). Then those decision alternatives which are not selected are eliminated from further consideration. On the decision-tree diagram, we draw a mark across the non-optimal decision branches, indicating that they are not to be followed.

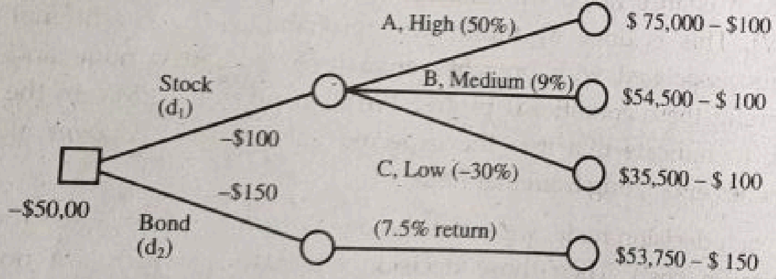
Example below illustrates how Bill could transform his investment problem into a tree format using numerical values.

Suppose Bill has $50,000 to invest in the financial market for one year.
His choices have been narrowed to two options:

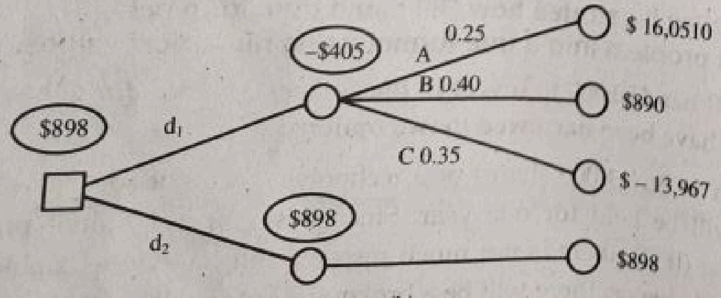
- **Option 1.** Buy 1,000 shares of a technology stock at $50 per share that will be held for one year. Since this is a new initial public offering (IPO), there is not much research information available on the stock; hence, there will be a brokerage fee of $100 for this size of transaction (for either buying or selling stocks). For simplicity, assume that the stock is expected to provide a return at any one of three different levels: a high level (A) with a 50% return ($25,000), a medium level (B) with a 9% return ($4,500), or a low level (C) with a 30% loss (- $1, 5000), Assume also that the probabilities of these occurrences are assessed at 0.25, 0.40, and 0.35, respectively. No stock dividend is anticipated for such a growth-oriented company.

- **Option 2.** Purchase a $50,000 U.S. Treasury bond, which pays interest at an effective annual rate of 7.5% ($3,750). The interest earned from the Treasury bond is nontaxable income. However, there is a $150 transaction fee for either buying or selling the bond. Bill's dilemma is which alternative to choose to maximize his financial gain. At this point, Bill is not concerned about seeking some professional advice on the stock before making a decision. We will assume that any long-term capital gains will be taxed at 20%. Bill's minimum attractive rate of return is known to be 5% after taxes. Determine the payoff amount at the tip of each branch.

Solution:



(a)



(b)

Figure: Decision tree for Bill's investment problem: (a) Relevant cash flows (before tax) and (b) net present worth for each decision path

Option 1

1. With a 50% return (a real winner) over a one - year holding period.
   - The net cash flow associated with this even at period 0 will include the amount of investment and the brokerage fee. Thus, we have

     Period 0: (– $50,000 – $ 100) = – $ 50,100
   - When you sell stock, you need to pay another brokerage fee. However, any investment expenses, such as brokerage commissions, must be included in determining the cost basis for investment. The taxable capital gains will be calculated as ($ 75,000 – $ 50,000 – $ 100 – $ 100) = $ 24,800. Therefore, the net cash flow at period 1 would be.

   Period 1: (+ $75,000 – $100) – 0.20 ($24,800) = $ 69,940
   - Then the conditional net present value of this stock transaction is

   PW (%) = – $50100 + $69,940 (P/F, 5%, 1) = $ 16,510

   This amount of $ 16,510 is entered at the trip of the corresponding branch. This procedure is repeated for each possible branch, and the resulting amounts are shown in figure (b)
2. With a 9% return, we
   - Period 0: – $ 50,100
   - Period 1 : $53,540
   - PW (5%) = – 50,100 +53,540 (P/F, 5%, 1) = $ 890
3. With a 30% loss, we
   - Period 0: – $ 50,100
   - Period 1 : $37,940
   - PW (5%) = – 50,100 +37,940 (P/F, 5%, 1) = – $ 13,967

Option 2: With no taxes in Bond and considering only brokerage commission

- Period 0: – $ 50,100

- Period 1 : $53,600
- PW (5%) = – 50,100 +53,600 (P/F, 5%, 1) = $ 898

Figure (b) of above shows the complete decision tree for Bill's investment problem. Now Bill can calculate the expected monetary value (EMV) at each chance node. The EMV of Option 1 represents the sum of products of the probabilities of high, medium, and low returns and the respective conditional profits (or losses):

EMV = $ 16,510 (0.25) + & 890(0.40) – $ 13,97 (0.35) = – $405

For Option 2, the EMV is simply

EMV = $ 898

In Figure 12.17, the expected monetary values are shown in the event nodes. Bill must choose which action to take, and this would be the one with the highest EMV, namely, option 2, with EMV = $898. This expected value is indicated in the tree by putting $898 in the decision node at the beginning of the tree. Note that the decision tree uses the idea of maximizing expected monetary value developed in the previous section. In addition, the mark || is drawn across the non-optimal decision problem (Option 1), indicating that it is not to be followed. In this simple example, the benefit of using a decision tree may not be evident. However, as the decision problem becomes more complex, the decision tree becomes more useful in organizing the information flow needed to make the decision. This is true in particular if bill must make a sequence of decisions, rather than a single decision, as well next illustrating.

Procedure for solving decision tree using PW analysis:

- Start at the top right of the tree.
- Determine PW for each outcome branch.
- Calculate expected value for each decision alternative.
- E (decision) = Σ (outcome estimate) × P (outcome)
- Select best decision
- Continue towards root decision in order to select best alternative.

Example:

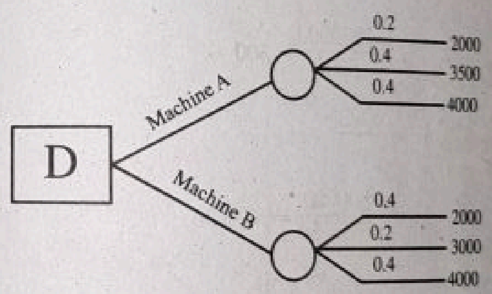
The income and corresponding probability of a machine A and machine B with Grade A, Grade B and Grade C, are provided as

	Machine A		Machine B	
	Probability	**Income**	**Probability**	**Income**
Grade C	0.2	2000	0.4	2000
Grade B	0.4	3500	0.2	3000
Grade A	0.4	4000	0.4	4000

The cost of machine A is 1050 and machine B is 1000. Find which machine should prefer?

Solution:

Decision tree



From the decision tree diagram,

Expected return Income from machine A

$= 0.2 \times 2000 + 0.4 \times 3500 + 0.4 \times 4000 = 3400$

Expected net profit from machine A = 3400 – 1050 = Rs. 2350

Expected return/income from machine B

$= 0.4 \times 2000 + 0.2 \times 3000 + 0.4 \times 4000 = 3000$

Expected net profit from machine B = 3000 – 1000 = Rs. 2000

Since, expected net profit from machine A is greater than that of machine B, the machine A should be prefer.

Old Question Solution

1. Calculate breakeven volume of a cable manufacturing company from the following data. Total cost = Rs. 1200000 Variable cost = Rs. 400000. Income from sales = Rs. 1500000 at production of 5000 unit.

(2069 Bhadra)

Solution:

Total cost (TC) = 1200000

Variable cost (VC) = 400000

Total cost = Fixed cost + variable cost

Fixed cost = 1200000 - 400000 = 800000

$$\text{Variable cost per unit (v)} = \frac{400000}{5000} = 80$$

$$\text{Sales per unit (s)} = \frac{1500000}{5000} = 300$$

$$\text{Now, Breakeven} = \frac{FC}{s - v}$$

$$= \frac{800000}{300 - 80} = 3636.36$$

∴ Breakeven volume = 3636.36 unit

2. A proposal is described by the following estimates: P = $20000, S = 0, N = 5 and net annual receipts = $7000. A rate of return of 20% is desired on such proposals. Construct a sensitivity graph of the life, annual receipts, and rate of return for deviations over a range of ± 20%. To which element is the decision most sensitive?

(2069 Bhadra)

Solution:

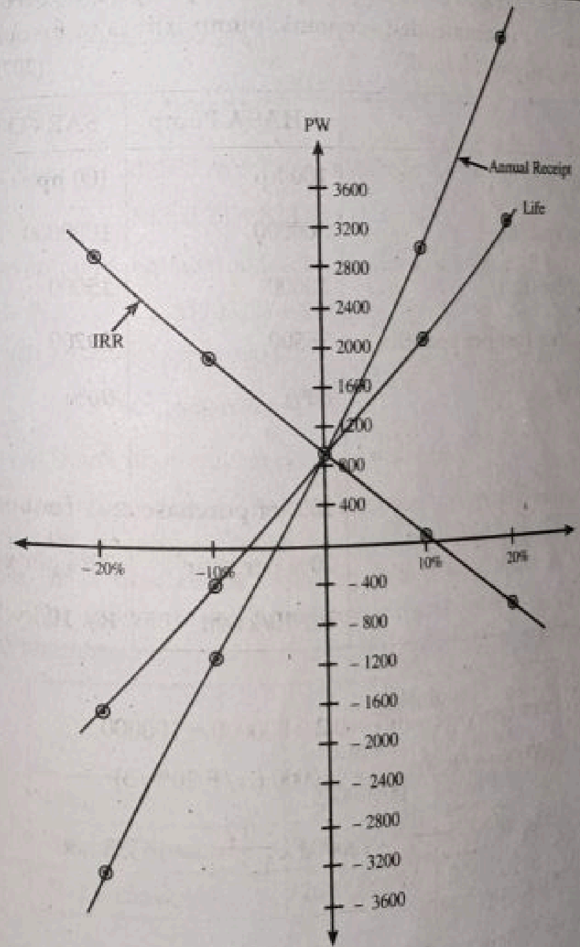
$$PW(20\%) = -20000 + 7000 \times \frac{1.2^5 - 1}{0.2 \times 1.2^5}$$

$$= 934.264$$

Now, variation in life, annual receipts & IRR are given in table which is obtain by a given equation.

$$PW(i\%) = -20000 + B \times \frac{(1+i)^N - 1}{i \times (1+i)^N}$$

Variation	10%	20%	-10%	-20%	Remarks
PW(N) Variation of life	2159.82	3278.57	-408.216	-1673.85	Annual receipts and IRR remains constant
PW(B) Variation of receipts	3027.713	5121.14	-1159.14	-3252.57	Life and IRR remains constant
PW (IRR) Variation of IRR	45.47	-782.3	1890.197	2920.055	Annual receipts & life remains constant



[Note:

Total variation also seen from calculated data

Life variation = 3278.57 – (–1878.81) = 5157.38

IRR variation = 2920.055 – (–782.30) = 3702.355

Benefit variation = 5121.14 – (–3252.57) = 8373.7 (more variable)]

∴ The maximum slope a from a graph is annual receipts. So, the project is most sensitive to annual receipts.

3. **Explain decision tree analysis.** (2070 Magh)

Refer at 6.4

4. **Calculate breakeven hours of operation per year to become cost equal and recommended economic pump if it is to be operated 5 hours daily at full load.** (2070 Magh)

	KHASA Pump	SARVO Pump
Capacity	100 hp	100 hp
Purchase cost (Rs.)	500000	1000000
Tax per year (Rs.)	10000	15000
Maintenance cost per year (Rs.)	36500	29200
Efficiency	80%	90%
Life year	5	5
Salvage value	20% of purchase cost for both	
MARR	20% per year	20% per year
Electricity cost	Rs. 10/kwhr	Rs. 10/kwhr

Solution:

For KHASA Pump, salvage value = $0.2 \times 500000 = 100000$

$$CR = 500000\,(A/P,20\%,5) - 100000\,(A/F,20\%,5)$$

$$= 500000 \times \frac{1.2^5 \times 0.2}{1.2^5 - 1} - 100000 \times \frac{0.2}{1.2^5 - 1} = 153751.88$$

Tax = 10000 per year

O & M = 36500 per year

$$\text{Operating expense of X hours} = \frac{(10 \times 100 \times 0.746 \times x)}{0.8} = 932.5\,x$$

$$\text{Total cost (TC)} = 932.5\,x + 10000 + 36500 + 153751.88$$

$$= 180826.22 + 932.5\,x \text{......(i)}$$

For SERVO Pump,

Salvage value = $0.2 \times 1000000 = 200000$

$$CR = 1000000\,(A/P,20\%,5) - 200000(A/F,20\%,5)$$

$$= 1000000 \times \frac{1.2^5 \times 0.2}{1.2^5 - 1} - 200000 \times \frac{0.2}{1.2^5 - 1} = 307503.76$$

Tax = 15000 per year

O & M = 29200 per year

$$\text{Operating expenses for X hours} = \frac{(10 \times 100 \times 0.746 \times x)}{0.9} = 828.88\,x$$

$$\text{Total cost} = 307503.76 + 15000 + 29200 + 828.88\,x$$

$$= 351703.76 + 828.88\,x \text{..........(II)}$$

For breakeven point, equate the total cost of two pump,

$$180826.22 + 932.5\,x = 351703.76 + 828.88\,x$$

$$103.62\,x = 170877.54$$

$$x = 1649.078 \text{ hrs/year}$$

∴ Break even hours of operation per year = 1649.078

5. **What are the sources of risk in engineering projects in Nepal? A real estate developer seeks to determine the most economical height for a new office building which will be sold after five years. The relevant net annual revenues and net resale values are as given below.** (2070 Bhadra)

	Height	
	4 floors	5 floors
First cost	125000000	200000000
Annual revenues	19910000	37815000
Net Resale value	200000000	300000000

The developer is uncertain about the interest rate i to use, but is certain that it is in the range of 5 to 30%. For each building height, find the range of values of i for which that building height is the most economical. Draw sensitivity diagram to support your answer.

Solution: *First part, Refer at 6.1*

Second part

For 4 floors building

$$PW = -125000000 + 19910000\ (P/A, i\%, 5) + 200000000\ (P/F, i\%, 5)$$

$$= -125000000 + 19910000 \times \frac{(1+i)^5 - 1}{i(1+i)^5} + \frac{200000000}{(1+i)^5}$$

Now varying i from 5% to 30%

i%	5	10	15	20	25	30
PW(i%)	117905113.8	74658829.19	41176755.06	14198602.11	-5920435.2	-22641991.5

From table,

PW	i%
14198602.11	20
-5920435.20	25
0	? (23.46) From interpolation

∴ The 4 floors building is economical for a ranges of 5% to 23.46% where PW is positive.

For 5 floors building

$$PW = -200000000 + 37815000 \times \frac{(1+i)^5 - 1}{i \times (1+i)^5} + \frac{300000000}{(1+i)^5}$$

Now varying i from 5 to 30%

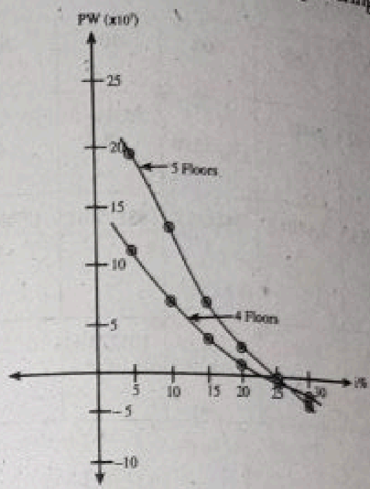
i%	5	10	15	20	25	30
PW(i%)	198777010.4	129624998.6	75914765.67	33653269.68	-876.80	-27100207.5

From table

PW	i%
33653269.68	20
-876.80	25
0	? (24.98) (By interpolation)

∴ 5 floors building is economical for a ranges of 5% to 24.98% where PW is positive.

Same result can be seen from sensitivity graph, economic interest rate is that where PW is positive.



6. **What are the sources of risk in engineering project in Nepal? Perform sensitivity analysis of the following project over a range of ± 30% in a) initial investment b) Net annual revenue and c) useful life. Draw also sensitivity diagram. Use PW formulation.**

Initial Investment (Rs.)	550000
Net Annual Revenue (Rs.)	150000
Salvage Value (Rs.)	80000
Useful life (years)	6
MARR	10%

(2071 Magh)

Solution:

First part refer at 6.1

$$PW = -550000 + 150000\ (P/A, 10\%, 6) + 80000\ (P/F, 10\%, 6)$$

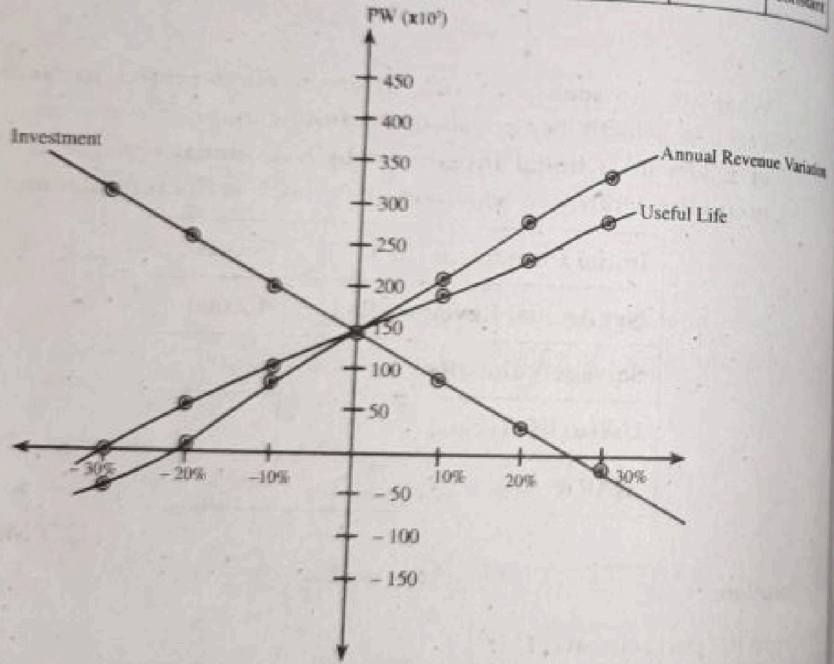
$$= -550000 + 150000 \times \frac{1.1^6 - 1}{1.1^6 \times 0.1} + \frac{80000}{1.1^6}$$

$$= 148447.02$$

Now at variable I, B and N

$$PW = -I + B\frac{1.1^N - 1}{1.1^N \times 0.1} + \frac{80}{1.1^N} \text{..........(i)}$$

Variation	10%	20%	30%	-10%	-20%	-30%	Remarks
PW(I), investment variation	93477.021	38447.02	-16552.98	203447.02	258447.02	313447.02	B & N remain constant
PW(B), Annual revenue variation	213775.93	279104.84	344433.75	83118.10	17789.19	-47539.71	I & N remain constant
PW(N), useful life variation	192998.7	235074.13	274810.93	101273.34	51323.35	-1566.33	I & B remain constant



The steeper slope is obtained for annual revenue.

∴ The most sensitive is the variation in Net annual revenue.

7. **A machine has a fixed cost of Rs. 4000000. It has variable cost Rs. 45000 per unit. Find BEP both in volume and the value if selling price per unit is Rs. 60000. What would be the effect on profit/loss when fixed cost increases by 10% and selling price decreases by 5%.** (2071 Magh)

Solution:

Fixed cost (FC) = 400000

Variable cost per unit (v) = 45000

Selling price per unit (s) = 60000

$$Q_{BEP} = \frac{FC}{s - v}$$

$$= \frac{4000000}{60000 - 45000}$$

$$= 266.67 \text{ unit}$$

In terms of volume i.e. sales

Q_{BEP} = 266.67×60000

= 16000000

Also, FC increases by 10%,

FC = 1.1×4000000

= 4400000

Selling price decrease by 5%

SP(s) = 0.95×60000

= 57000 per unit

Total cost = FC + variable cost

= $4400000 + 266.67 \times 45000$

= 16400000

Sales = 266.67×57000

= 15200000

Cost > Sales

$$\therefore \text{Loss \%} = \frac{16400000 - 15200000}{16400000}$$

= 7.4% of total cost

8. **Explain the decision tree analysis**

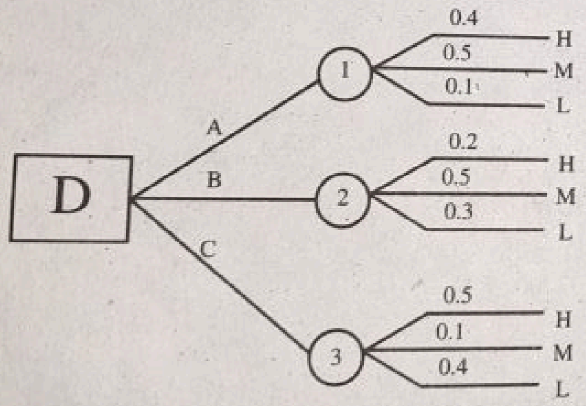
Refer at 6.4

9. **For the improvement of a manufacturing plant, following three alternatives are being considered. The estimated investments and the corresponding increment in income are also given as below. Draw decision tree diagram of the situation and decide on the best alternative using FW formulation. MARR = 15%, life of the project is 6 years.**

Alternatives	Investment Cost	Sales	Probability	Annual income
A	1000000	High success	0.4	500000
		Medium success	0.5	300000
		Low success	0.1	125500
B	600000	High success	0.2	400000
		Medium success	0.5	250000
		Low success	0.3	100000
C	400000	High success	0.5	200000
		Medium success	0.1	125000
		Low success	0.4	50000

(2071 Bhadra)

Solution:



H = High Success
M = Medium Success
L = Low Success

Annual incomes are obtained by multiplying incomes of different success with their probability

For alternative A = 500000 × 0.4 + 300000 × 0.5 + 125500 × 0.1 = 362500

For alternative B = 400000 × 0.2 + 250000 × 0.5 + 100000 × 0.3 = 235000

For alternative C = 200000 × 0.5 + 125000 × 0.1 + 50000 × 0.4 = 132500

Now, PW for Alternative A = –1000000 + 362500 (P/A, 15%, 6)

$$PW_A = -1000000 + 362500 \times \frac{1.15^6 - 1}{1.15^6 \times 0.15}$$

$$= 371874.97$$

PW for Alternative B,

$$PW_B = -600000 + 235000 \times \frac{1.15^6 - 1}{1.15^6 \times 0.15}$$

$$= 289340$$

PW for Alternative C,

$$PW_C = -400000 + 132500 \times \frac{1.15^6 - 1}{1.15^6 \times 0.15}$$

$$= 101388$$

Hence PW_A is greater than other, thus, alternative A is most feasible.

10. **Perform sensitivity analysis of the following project over a range of 10 to 50 percent in**

i) **Initial investment and**

(ii) **MARR using PW formulation. Assume Sv = 0. Draw sensitivity diagram also.**

Initial investment = 400000

Net annual income = 80000

Life = 12 years, MARR = 15% (2071 Bhadra)

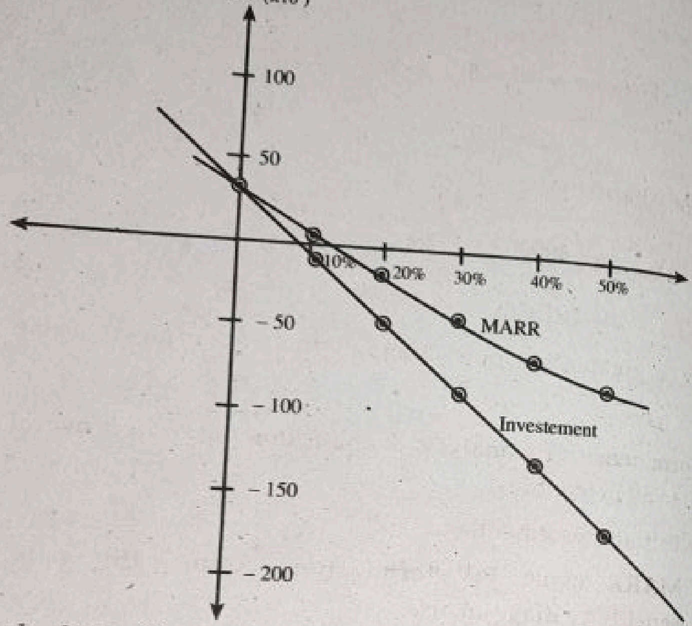
Solution: writing a PW directly,

$$PW = -400000 + 80000 \times \frac{1.15^{12} - 1}{1.15^{12} \times 0.15} = 33650$$

In variation of initial investment and MARR

$$PW = -I + 80000 \times \frac{(1+i)^{12} - 1}{(1+i)^{12} \times i} \quad \text{............(i)}$$

Variation	10%	20%	30%	40%	50%	Remarks
PW (I): Investment in variation	-6350.48	-46350.48	-86350.48	-126350.48	-166350.48	MARR remains constant
PW (i%) MARR variation	7278.27	-16542	-38120.7	-57724.0	-75581.3	Investment remains constant



From graph, slope of investment > Slope of MARR

∴ Investment is more sensitive.

11. A company products an electronics timing switch that is used in consumer and commercial products made by several other manufacturing firms. The fixed cost and total cost are Rs. 40000 and Rs. 85000 respectively. The total sales are Rs. 105000 and sales volume is 15000 for this situation.

i) Find the breakeven point in terms of number of unit.

ii) What should be the output if the profit desired is Rs.50000?

(2072 Ashwin)

Solution:

i) Total cost (TC) = Fixed cost (FC) + variable cost (VC)

VC = TC – FC = 85000 – 40000 = 45000

Variable cost per unit (v) $= \frac{45000}{15000} =$ Rs. 3 per unit

Selling cost per unit (s) $= \frac{105000}{15000} =$ Rs. 7 per unit.

$$Q_{BEP} = \frac{FC}{s - v}$$

$$= \frac{40000}{7 - 3}$$

$$= 10000 \text{ units.}$$

ii) If the profit desired is Rs. 50000

Profit = Total sales – Total cost

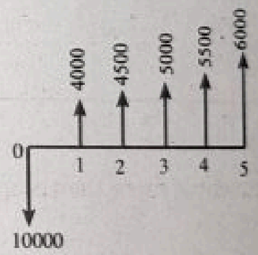
$50000 = s \times Q - (40000 + v \times Q)$

or, $90000 = (7 - 3) \times Q$

or, $Q = 22500$

Break even units for the profit to be Rs. 50000 is 22500

12. Draw sensitivity chart using PW formulation of the following cash information's. It is desired to evaluate the sensitivity of PW to ± 30% change on: i) interest ii) investment. (2072 Ashwin)



Solution:

Assume MARR = 15%

$$PW = -10000 + 4000\,(P/A, 15\%\ 5) + G\,(P/G, 15\%, 5)$$

$$= -10000 + 4000 \times \frac{1.15^5 - 1}{1.15^5 \times 0.15} + \frac{500}{0.15^2} \times \frac{(1.15^5 - 5 \times 0.15 - 1)}{1.15^5}$$

$$= 6292.19$$

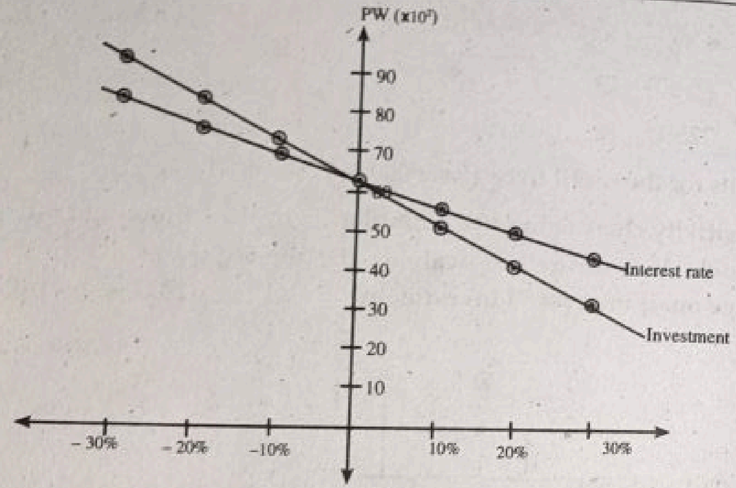
PW in variation of I & i

$$PW = -I + 4000 \times \frac{(1+i)^5 - 1}{(1+i)^5 \times i} + \frac{500}{i^2} \times \left[\frac{(1+i)^5 - 5i - 1}{(1+i)^5}\right]$$

or, this can be written as

$$PW = -I + \frac{4000}{(1+i)} + \frac{4500}{(1+i)^2} + \frac{5000}{(1+i)^3} + \frac{5500}{(1+i)^4} + \frac{6000}{(1+i)^5}$$

Variation	10%	20%	30%	-10%	-20%	-30%	Remarks
PW (i) Interest variation	5692.97	5124.3	4587.67	6936.72	7617.6	8342.18	Investment remains constant
PW (I) Investment variation	5296.19	4296.19	3296.19	7296.19	8296.19	9296.19	Interest remain constant



From graph, slope of investment is more than slope of interest rate, thus investment is more sensitivity.

13. Explain about the decision tree analysis. Perform sensitivity analysis of the following project over range of ± 30% at the interval of ± 10% in

i) Initial investment (ii) Net Annual Revenue of (iii) Useful life

USE PW Formulation

Initial Investment (Rs.)	1,00,000
Net Annual Revenue (Rs.)	40,000
Salvage value (Rs.)	15,000
Useful life (years)	6
MARR (%)	10

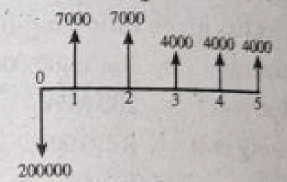
(2073 Bhadra)

Solution:

Decision Tree Analysis : Alternative evaluation may require a series of decision where the outcome from one stage is important to the next stage of decision making. When each alternative is clearly defined and probability estimates can be made to account for risk, it is helpful to perform the evaluation using decision tree. Decision tree is a powerful means of facilitating the analysis of important problems, especially those that involve sequential decisions and variable outcomes over time.

[For detail Refer 6.4] [For Second Part [Procedure is same as "2071 Magh"]

14. Consider the following cash flow diagram. Plot changes in present worth to ±20% and ±30% for the project life. Let MARR = 10%. Assume salvage value = 0. **(2073 Magh)**



Solution:

$$PW(10\%) = -20000 + \frac{7000}{(1+0.1)^1} + \frac{7000}{(1+01)2} + \frac{4000}{(1+0.1)^3} + \frac{4000}{(1+0.1)^4} + \frac{4000}{(1+0.1)^5}$$

$= 369.75$

[Note: This can be solve by use AW Formulation, Annually of 7000 for 2 years and 4000 for 3 years]

For + 20%N

$$PW = -2000 + \frac{7000}{1.1^{1.2}} + \frac{7000}{1.1^{2.4}} + \frac{4000}{1.1^{3.6}} + \frac{4000}{1.1^{4.8}} + \frac{4000}{1.1^{6}}$$

$= 560.19$

For - 20%N

$$PW = -2000 + \frac{7000}{1.1^{0.8}} + \frac{7000}{1.1^{1.6}} + \frac{4000}{1.1^{2.4}} + \frac{4000}{2.1^{3.2}} + \frac{4000}{1.1^{4}} = 1358.73$$

For + 30% N

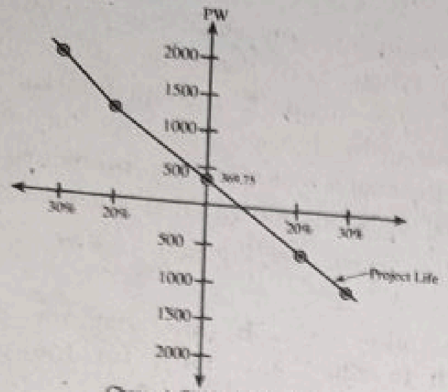
$$PW = -2000 + \frac{7000}{1.1^{1.3}} + \frac{7000}{1.1^{2.6}} + \frac{4000}{1.1^{3.9}} + \frac{4000}{1.1^{5.2}} + \frac{4000}{1.1^{6.5}}$$

$= -1004.339$

For - 30% N

$$PW = -2000 + \frac{7000}{1.1^{0.7}} + \frac{7000}{1.1^{1.4}} + \frac{4000}{1.1^{2.1}} + \frac{4000}{1.1^{2.8}} + \frac{4000}{1.1^{3.5}}$$

$= 1876.73$



Change in PW for project life

15. **A small hydro project has given following information**

Initial investment	=	**Rs. 10,00,000**
Energy Generated for year	=	**200 MW**
Annual repairing cost	=	**Rs. 6,000,000**
Market Price of energy	=	**Rs. 4/Kwhr**
Salvage value	=	**Rs. 1,00,000**
MARR	=	**10%**

Find its breakeven point of time. **(2073 Magh)**

Solution: Breakeven point is the point at which revenue is exactly equal to costs.

Initial investment = 10,00,000

Energy generated = 200 MW/year = 200×10^3 kw/(365 × 24) hrs.

$$\text{Market price of energy i.e. Revenue} = \frac{200 \times 10^3 \times 4}{365 \times 24} = \text{Rs. } 91.32 \text{ per hrs.}$$

Revenue per year = 91.32 × 24 × 365 = 799963.2

For BEP equating revenue and cost

Variation	+ 40%	- 40%	Remarks
(i) PW (I) Investment Variation	19601.19	119601.19	A, I an N remain constant.
(ii) PW(A) Annual revenue variation	72084.86	67117.49	I, I and N remain constant
iii) PW (N) useful life variation	26184.14	-42881.29	I, i and A remain constant
iv) PW(i) MARR(i) variation	-16813.88	8843.49	I, A and N remain (constant)

Revenue = Cost

$$-10{,}00{,}000\,(1 + 0.1)^N - (6{,}00{,}000 - 799963.2)\frac{(1.1^N - 1)}{0.1} + 1{,}00{,}000 = 0$$

N = 6.736 years

Hence, Break even period = 6.736 years

16. **A project cost Rs. 125,000 with Annual revenue of Rs. 65,000 and annual cost of Rs. 35,000. Salvage value will be 8 % of the initial investment. Perform sensitivity analysis using PW formulation over a range of ±40% in (i) Initial investment ii) Annual revenue iii) Useful life and iv) MAAR. Draw the sensitivity diagram and indicate the most sensitive and least sensitive parameters.**

(2074 Bhadra)

Solution: Here, I = 125000, MARR = 10% (assumed)

Net annual revenue (A) = 65000 - 35000

= 30000

S = 8 % of I

= 10,000

Useful life is not given so by simple payback period

$$N = \frac{I}{A} = \frac{125000}{30000}$$

= 4.16

So, take N = 5 years.

Now, PW = - 125000 + 30000 (p/A 10%5) + 10000 (p/f 10%, 5) ....(a)

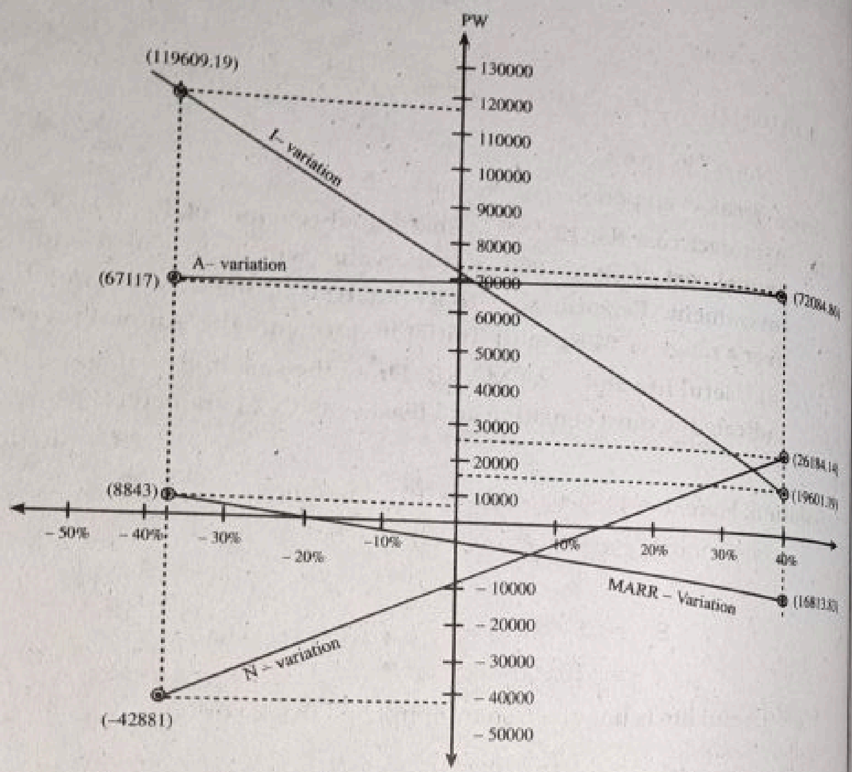
$$= -125000 + 30000 \times \frac{1.1^5 - 1}{0.1 \times 1.1^5} + \frac{10000}{1.15}$$

= Rs. 69601 .19

In terms of variable I, A, N and i

$$PW = I + A\left[\frac{(1.i)^N - 1}{i \times (1+i)^N}\right] + \frac{10000}{(1+i)^N}$$

For Investment variation increase (I) by 40% and decrease (I) by 40% i.e. I = 125000 × 1.4 = 175000 and I = 125000 × 0.6 = 75000 and keeping all other variables same as in equation (a) calculated two value of pw for I = 175000 and PW for I = 75000. And follow same procedure for variation in other variables also then in tabular form:



From graph we see Investment variation has steeper slope.

∴ The most sensitive is the variation in Investment.

Note: To determine steepest slope you can find out the difference between + 40% value and – 40% of PW for all cases, then the case having highest value must have steepest slope.

17. Define breakeven point and break even volume. How does interest rate change affect the project? **(2074 Bhadra)**

Solution: *Refer 6.2.2*

18. Explain the concept of scenario and decision tree analysis. If 20 watt CFL bulb Price is Rs. 280 and 100 watt filament bulb price is Rs. 30 at market but their lighting power is equal. Which bulb do you prefer to use in your house when electricity cost is Rs. 12 per unit? **(2075 Baishakh)**

Solution: *For theory refer 6.2.3 and 6.4*

From Given, For CFL bulb of 20 watt:

Investment = Rs. 280

Cost of use = $\left(\frac{20}{1000}\right) \times x \times 12 = 0.24\,x$

Above x is the no. of hours of lighting

Total cost = $280 + 0.24\,x$

For filament bulb of 100 watt:

Investment (I) = 30

Cost of use = $\left(\frac{100}{1000}\right) \times x \times 12 = 1.2\,x$

Total cost = $30 + 1.2x$

At breakeven point

Total cost of bulb = Total cost of filament bulb

$280 + 0.24\,x = 30 + 1.2x$

At breakeven point

Total of bulb = Total cost of filament bulb

$280 + 0.24\,x = 30 + 1.2\,x$

On solving: $x = 260.42$ hours

Since, I have to bulb for more than 260.42 hours and the cost of use of CFL is 0.24x, i.e. for higher value of x, cost of use for CFL is less than filament. So I prefer CFL bulb.

19. Perform sensitivity analysis for the following project over a range of ± 30% in parameters: (i) Initial investment (ii) Annual revenue (iii) Life year.

I	Rs. 500,000
A	Rs.120,000
Salvage	Rs. 80,000
Life year	6 year
MARR	10% per year

(2075 Baishakh)

Solution: *Refer 2071 Magh in (QN.1)*

20. Perform a sensitivity analysis (Over a range of (15% with 5% increment) with using IRR (AW formulation) among the parameters (i) useful life (ii) Initial Investment (iii) Revenues. It is given that, I = Rs 1,00,000 SV = RS 22,000 O&M = Rs 12,000

Revenues = RS 40,000 Useful life = 6 years and MARR = 10% Suggest which of the considered parameter of the project is more sensitive and why? (2076 Bhadra)

Solution: *Refer solution of 2071 Magh in (QN.7)*

21. What are the different tools used in economics to assess the risk of any project? Explain each of them briefly. (2076 Bhadra)

Solution: *Refer 6.3.*

Extra Question Solution

1. CFL bulb of 20 watt costs Rs. 250 where as Filament bulb of 100 watt costs Rs. 30 which bulb do you prefer in your house? Why? Electricity cost is Rs. 10 per unit (Kwh). Assume light the bulb in a house is 4.5 hours per day.

Solution:

For CFL bulb:

Initial cost = Rs. 250

Cost of use $= \left(\frac{20}{1000}\right) \times x \times 10 = 0.2x$

Where x is the number of hours lighting the bulb

Total cost = 250 + 0.2 x

For Filament bulb

Initial cost = 30

Cost of use $= \left(\frac{100}{1000}\right) \times x \times 10 = x$

Total cost (TC) = 30 + x

At breakeven point,

TC of CFC bulb = TC of Filament bulb

250 + 0.2 x = 30 + x

$x = \frac{220}{0.8} = 275$ hours

In a day, light the bulb = 4.5

Therefore, $x = \frac{275}{4.5} = 61.11 \approx 62$ days

Since I have to use for more than 275 hours (62 days), I will use CFL bulb in my house.

2. Perform sensitivity analysis using IRR and BCR (with increment of 10%) over a range of ± 20 % in a) initial investment and b) Net annual revenue.

Initial investment (I) = Rs. 2,00,000
Annual Revenues (R) = Rs. 50,000
Annual Expense (E) = Rs. 5,000
Salvage value (SV) = Rs. 25,000
Useful life (N) = 10 years, MARR = 12% per year

Solution: Criteria of merit: IRR

Using AW formulation,

- 2,00,000 (A/P,i%,10) + (50,000 – 5000) + 25,000 (A/F,i%,10) = 0

or, $-200000\frac{(1+i)^{10} \times i}{(1+i)^{10}-1} + 45,000 + 25,000 \times \frac{i}{(1+i)^{10}-1} = 0$

IRR = 18.95%

When the capital investment varies by ± 20%

At 10%,

$AW = -2,00,000 \times 1.1 \times \frac{(1+i)^{10} \times i}{(1+i)^{10}-1} + 45,000 + 25,000 \times \frac{i}{(1+i)^{10}-1}$

= 0

IRR = 16.37 %

At 20%,

$AW = -2,00,000 \times 1.2 \times \frac{(1+i)^{10} \times i}{(1+i)^{10}-1} + 45,000 + 25,000 \times \frac{i}{(1+i)^{10}-1} =$

0

IRR = 14.15 %

At – 10%,

$AW = -2,00,000 \times 0.9 \times \frac{(1+i)^{10} \times i}{(1+i)^{10}-1} + 45,000 + 25,000 \times \frac{i}{(1+i)^{10}-1}$

IRR = 21.99 %

Similarly, At – 20 %, IRR = 25.67 %

When net annual revenue (R – E) varies ± 20 %

R – E = 50000 – 5000 = 45000

At 10 %, $AW = -2,00,000 \times \frac{(1+i)^{10} \times i}{(1+i)^{10}-1} + 45,000 \times 1.1 + \frac{25000 \times i}{(1+i)^{10}-1} = 0$

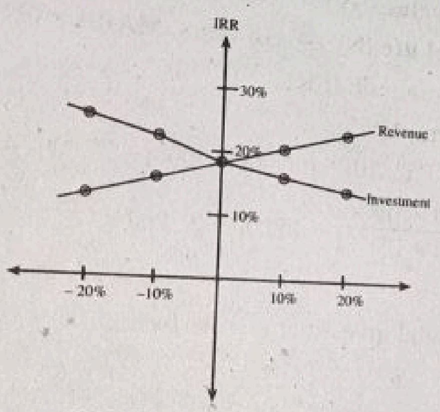
IRR = 20.66 %

Similarly, At 20 %, IRR = 23.36 %

At – 10%, $AW = -2,00,000 \frac{(1+i)^{10} \times i}{(1+i)^{10}-1} + 45000 \times 0.9 + \frac{25000 \times i}{(1+i)^{10}-1} = 0$

IRR = 16.18 %

Similarly, At – 20 %, IRR = 13.32 %



Different in investment = 25.67 – 14.15 = 11.52 %

Different in revenue = 23.36 – 13.32 = 10.64 %

∴ Investment is more sensitivity.

Criteria of merit: BCR

Using FW Formulation,

$$BCR_{modified} = \frac{(50000 - 5000) \times (F/A, 12\ \%, 10)}{200000\ (F/P, 12\ \%, 10) - 25000}$$

$$= \frac{45000 \times (1.12^{10} - 1)/0.12}{200000 \times 1.12^{10} - 25000} = 1.32$$

When the investment (I) varies by ± 20%

At 10 %, BCR $= \frac{45000 \times (F/A, 12\ \%, 10)}{200000 \times 1.1\ (F/P, 12\ \%, 10) - 25000} = 1.19$

Similarly, At 20%, BCR = 1.09

At – 10%, BCR $= \frac{45000 \times (F/A, 12\%, 10)}{200000 \times 0.9\ (F/P, 12\%, 10) - 25000} = 1.47$

Similarly,

At – 20%, BCR = 1.67

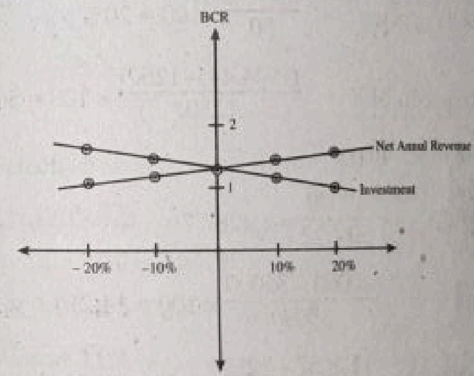
Where net annual revenue varies by ± 20%

At 10%, BCR $= \frac{4500 \times 1.1\ (F/A, 12\%, 10)}{200000\ (F/P, 12\%, 10) - 25000} = 1.45$

At 20%, BCR $= \frac{4500 \times 1.2\ (F/A, 12\%, 10)}{200000\ (F/P, 12\%, 10) - 25000} = 1.58$

At –10%, BCR $= \frac{4500 \times 0.9\ (F/A, 12\%, 10)}{200000\ (F/P, 12\%, 10) - 25000} = 1.19$

Similarly, At – 20%, BCR = 1.05



Different in investment = 1.67 – 1.09 = 0.58

Different in revenue = 1.58 – 1.05 = 0.53

More slope in investment. Hence investment is more sensitivity.

3. **Perform a sensitivity analysis by breakeven analysis under the following data.**

Fixed cost	=	Rs. 35,000
Variable cost	=	Rs. 66,000
Output per unit	=	Rs. 3,000
Sales revenue per unit	=	Rs. 1,50,000

a) Selling price comes down to Rs. 40

b) Fixed cost increase to Rs. 40,000

c) Variable cost increase by 10%

Solution:

$$\text{Breakeven point (BEP)} = \frac{\text{Fixed cost}}{\text{(Selling price per unit – variable cost per unit)}}$$

Variable cost per unit = 66,000/3,000 = Rs. 22

Selling cost per unit = 1,50,000/3,000 = Rs. 50

$$\therefore \text{BEP} = \frac{35000}{50-22} = 1{,}250 \text{ units}$$

a) Selling price down to Rs. 40

$$\text{BEP} = \frac{35000}{40-22} = 1{,}944.44 \text{ units}$$

$$\text{Reduction in selling price} = \frac{50-40}{50} \times 100 = 20\%$$

$$\text{Increase in BEP} = \frac{(1944.44-1250)}{1250} \times 100 = 56\%$$

b) Cost increase to Rs. 40,000

$$\text{BEP} = \frac{40000}{50-22} = 1{,}428.57$$

$$\text{Increase in FC} = \frac{40000-35000}{35000} \times 100 = 14.21\%$$

$$\text{Increase in BEP} = \frac{1428.57-1250}{1250} \times 100 = 14.28\%$$

c) Variable cost increase by 10%

i.e. variable cost = $1.1 \times 22 = 24.2$

$$\text{BEP} = \frac{35000}{50-24.2} = 1356.58 \text{ units}$$

$$\text{Increase in variable cost} = \frac{(24.2-22)}{22} \times 100 = 10\%$$

$$\text{Increase in BEP} = \frac{(1356.58-1250)}{1250} \times 100 = 8.5\%$$

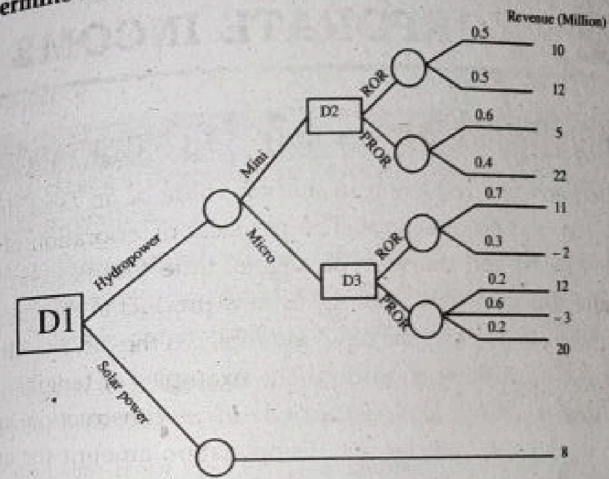
Results;

- 20%, increase in selling price results in 56% increase of BEP

- 14.21% increase in FC results in 14.28% increase in BEP
- 10% increase in variable cost results in 8.5% increase of BEP

Out of three factors (Selling price, fixed cost, variable cost) BEP is more sensitive in selling price.

4. **Determine the best decision at the decision node D1.**



Solution: Selection step by step from right to left.

Decision at node D_2

$ROR = 0.5 \times 10 + 0.5 \times 12 = 11$

$PROR = 0.6 \times 5 + 0.4 \times 22 = 11.8$

Select PROR as 11.8

Decision at node D_2

$ROR = 0.7 \times 11 + 0.3 \times (-2) = 7.1$

$PROR = 0.2 \times 12 + 0.6 \times (-3) + 0.2 \times 20 = 4.6$

Select ROR as 7.1

Selection at D_1.

From hydropower = $11.8 \times 0.4 + 7.1 \times 0.6 = 8.98$

From solar power = 8

Therefore, select hydropower plant as higher revenue of 8.98 million.

7 DEPRECIATION & CORPORATE INCOME TAX

7.1 Concept and Terminology of Depreciation

It represents the reduction in market value of an asset due to age, wear and tear and obsolescence. The physical deterioration of the asset occurs due to wear and tear with passage of time. Obsolescence occurs due to availability of new technology or new product in the market that is superior to the old one and the new one replaces the old even though the old one is still in working condition. The examples of tangible assets for which the depreciation analysis is carried out are construction equipment, buildings, machinery, vehicles, etc. Depreciation amount for an asset is usually calculated on yearly basis. Depreciation is considered as expenditure in the cash flow of the asset, although there is no physical cash outflow.

More specially, depreciation is an accounting concept that establishes an annual deduction against before-tax income such that the effect of time and use on an asset's value can be reflected in a firm's financial statements.

Depreciation affects the income tax to be paid by an individual or a firm as it is considered as an allowable deduction in calculating the taxable income. Generally the income tax is paid on taxable income which is equal to gross income less the allowable deductions (expenditures). Depreciation reduces the taxable income and hence results in lowering the income tax to be paid.

The actual amount of depreciation can never be established until the asset is retired from service.

Basic requirements of depreciation

1. It must be used in business or held to produce income.
2. It must have a determinable useful life and the life must be longer

than one year.

3. It must be something that wears out, decays, gets used up, becomes obsolete or losses value from natural causes.
4. It is not inventory, stock in trade or investment property.

Causes of depreciation

1. Wear and Tear
2. Effusion of time
3. Obsolescence
4. Exhaustion or depletion
5. Accident
6. Fall in market value

We can classify depreciation into the categories of physical or functional depreciation.

Physical depreciation can be defined as a reduction in an asset's capacity to perform its intended service due to physical impairment. Physical depreciation can occur in any fixed asset in the form of

(1) Deterioration from interaction with the environment, including such agents as corrosion, rotting, and other chemical changes, and

(2) Wear and tear from use. Physical depreciation leads to a decline in performance and high maintenance costs.

Functional depreciation occurs as a result of changes in the organization or in technology that decrease or eliminate the need for an asset. Examples of functional depreciation include obsolescence attributable to advances in technology, a declining need for the services performed by an asset, and the inability to meet increased quantity or quality demands.

Terminologies of depreciation

Before discussing about different methods of depreciation, it is necessary to know the common terms used in depreciation analysis.

1. **Book value:** Represent the remaining, un-depreciated capital investment on the books determined at the end of each year. It is the value of asset recorded on the accounting books of the firm at a given time period. It is generally calculated at the end of each year. Book value at the end of a given year equals the initial cost less the total depreciation amount till that year.
2. **Unadjusted cost or Initial Cost:** Initial cost is the total cost of acquiring the asset. It includes the cost of the assets including purchase price, delivery, and installation fees etc.

3. **Recovery period:** Depreciable life 'n' of the assets in years.
4. **Salvage value (SV):** Salvage value represents estimated market value of the asset at the end of its useful life. It is the expected cash inflow that the owner of the asset will receive by disposing it at the end of useful life.
5. **Market value:** The amount that will be paid by the willing buyer to willing seller for a property where each has equal advantage and no compulsion to buy or sell.
6. **Useful life:** It represents the expected number of years the asset is useful in terms of generating revenue. The asset may still be in working condition after the useful life but it may not be economical. Useful life is also known as depreciable life. The asset is depreciated over its useful life.

7.2 Basic methods of depreciation

7.2.1 Straight line method

In this method the property is assumed to lose value by a constant amount every year, and thus a fixed amount of original cost is written off every year so that at the end of the term when the asset is worn out, only the scrap value remains. In other words the book value of the asset decreases at a linear rate with the time period.

It assumes that a constant amount is depreciated each year over the depreciable life of the asset.

Mathematically,

$$D_n = \frac{(I - S)}{N}$$

Where, D_n = Annual depreciation deduction in the year N.

I = Initial cost

S = Salvage value

N = Life of the asset

Example: A machine costing of Rs. 4,000 is estimated to have life 10 years, and salvage value is 1,000. Find

i) Annual depreciation **ii) Rate of depreciation**

iii) Depreciation amount for the 6th year

iv) Accumulated depreciation and book value at end of 6th year

Solution:

$$\text{Annual depreciation} = \frac{4000 - 1000}{10} = \text{Rs. 300 per year}$$

$$\text{Rate of depreciation} = \frac{300}{3000} \times 100 = 10\% \text{ per year}$$

$$\text{OR, Rate of depreciation} = \frac{1}{N} \times 100 = \frac{1}{10} \times 100 = 10\% \text{ year}$$

iii) Depreciation for 6th year = Annual depreciation = Rs. 300

iv) Accumulated depreciation amount for the 6th year $= \frac{(4000 - 1000)}{10} \times 6$

$= 3000 \times 6 = \text{Rs. } 1800$

Book value at end of 6th year = 4000 – 1800 = Rs. 2200

7.2.2 Diminishing/Declining Balance Method/ Matheson Method / Constant percentage method

It is an accelerated depreciation method. In this method the annual depreciation is expressed as a fixed percentage of the book value at the beginning of the year and is calculated by multiplying the book value at the beginning of each year with a fixed percentage. Thus this method is also sometimes known as fixed percentage method of depreciation. The ratio of depreciation amount in a given year to the book value at the beginning of that year is constant for all the years of useful life of the asset. When this ratio is twice the straight-line depreciation rate, the method is known as double-declining balance (DDB) method. In other words the depreciation rate is 200% of the straight-line depreciation rate. Double-declining balance (DDB) method is the most commonly used declining balance method.

It is assumed that the annual cost of depreciation is fixed percentage of the book value (BV) at the beginning of the year. The ratio of the depreciation in any one year to the BV at the beginning of the year is constant throughout the life of the asset.

Mathematically,

Depreciation at end of first year $D_1 = \alpha I$

Where I = initial cost, α = depreciation ratio

At the end of 2nd year $= \alpha(I - D_1)$

$= \alpha(I - \alpha I) = \alpha I(1 - \alpha)$

At the end of 3rd year = $\alpha(1 - D_1 - D_2) = \alpha I (1 - \alpha)^2$

For any year 'n'

$$D_n = \alpha I (1 - \alpha)^{n-1}$$

Book value = $I (1 - \alpha)^n$

α = Declining depreciation rate = 2 × straight line depreciation rate.

Issues Regarding salvage value

- Salvage value must be estimated at the beginning of depreciation analysis.
- If final book value (B_N) doesn't equal to estimated salvage value (S), we have to make the adjustment in our depreciation analysis method.

Case I $B_N > S$

To reduce Final book value (B_N) to salvage value by switching from declining balance to straight line.

When $B_N > S$, we are faced with a situation in which we have not depreciated the entire cost of the asset and thus have not taken full advantage of depreciation's tax deferring benefits. If you would prefer to reduce the book value of an asset to its salvage value as quickly as possible, it can be done by switching from DB to SL whenever SL depreciation results in larger depreciation charges and therefore a more rapid reduction in the book value of the asset. The switch from DB to SL depreciation can take place in any of the *n* years, the objective being to identify the optimal year to switch. The switching rule is as follows: If depreciation by DB in any year is less than (or equal to) what it would be by SL, we should switch to and remain with the SL method for the duration of the project's depreciable life. The straight-line depreciation in any year *n* is calculated by

Switching rule : If depreciation by declining balance in any year is less than or equal to the depreciation by straight line, then we would switch to and remain with straight line for the duration of the project's depreciable life.

Straight line depreciation in any year n (D_n)

$$D_n = \frac{\text{book value at the beginning of year n - salvage value}}{\text{Remaining useful life at beginning of year n}}$$

Case II $B_N < S$

With a relatively high salvage value, it is possible that the book value of the asset could decline below the estimated salvage value. When $B_N < S$ we must readjust our analysis because tax law does not permit us to depreciate assets below their salvage value. To avoid deducting depreciation charges that would drop the book value below the salvage value, you simply stop depreciating the asset whenever you get down to $B_N = S$. In other words, if, at any period, the implied book value is lower than S, then the depreciation amounts are adjusted so that $B_N = S$.

i.e. If the book value is lower than S, at any period, then depreciation amount are adjusted as $B_N = S$.

Note:

1. **Double declining balancing method:**

   Depreciation rate, $R = \frac{100}{N} \times 2\ \%$

   Where, N = useful life

2. **Declining balance/ Diminishing balance method:**

   Depreciation rate, $R = 1 - \left(\frac{Sv}{P}\right)^{\frac{1}{N}}$

   Where, Sv = salvage value

   P = original cost

   If = Sv = 0, This formula cannot be applied.

Example: Consider the following accounting information for a computer system.

Cost basis of the asset, I	=	**Rs. 20,000**
Useful life (N)	=	**5 years**
i) Salvage value (S)	=	**Rs. 500**
ii) Salvage value	=	**Rs. 2000**

Solution:

Declining balance rate (α) = $\frac{1}{5} \times 2 = 40\%$

Calculating the depreciation by Double Decline Balance (DDB) for each year

N	B_{N-1}	$D_N = \alpha \times B_{N-1} = 0.4 \times B_{N-1}$	$B_N = B_{N-1} - D_N$
1	20000	8000	12000
2	12000	4800	7200
3	7200	2880	4320
4	4320	1728	2592
5	2592	1036.8	1555.2

i) Here book value is Rs. 1552.2 at the end of 5 which is greater than Rs. 500. Therefore we use the switching.

So, compute the SL depreciation for each year and compare with DDB and use the decision rule.

N	SL Depreciation	DB depreciation	Decision
1	(20000 – 500)/5 = 3900	< 8000	Do not switch
2	(12000 – 500)/4 = 2875	< 4800	Do not switch
3	(7200 – 500)/3 = 2233.33	< 2880	Do not switch
4	(4320 – 500)/2 = 1910	> 1728	Switch to SL

The depreciation schedule is

N	DDB with switching to SL	End of year Book value (Rs.)
1	8000	12000
2	4800	7200
3	2880	4320
4	1910	2410
5	1910	500

ii) Book value < Salvage value (2000)

Depreciation amount are adjusted as $B_N = S$

N	D_N	B_N (at end of year)
1	8000	12000 > 2000
2	4800	7200 > 2000
3	2880	4320 > 2000
4	1728	2592 > 2000
5	1036.8 > 592	2000 (2592 – 592)

7.2.3 Sinking Fund method

Sinking fund is an amount which has to be set aside at fixed intervals of time (say actually) out of the gross income so that at the end of the useful life of the building or property, the fund should accumulate to the initial cost of the property. A building, a machine, a vehicle etc., becomes useless after certain years i.e. at the end of its life. Hence it is necessary to make some provision whereby the owner can accumulate to a sum required for rebuilding the premises or can replace the article. For the above purpose sinking fund is periodically collected and deposited in a bank to get highest compound interest or sinking fund insurance policy is contracted with the insurance company throughout the life of building or article. In case when a building is built up or a vehicle is purchased by taking loan, a small portion of rent or income is set aside every year or at regular intervals and may be paid directly to the lender by way of installments.

The fixed sum depreciated at the end of every time period earns an interest rate of i% compounded annually.

In this method, the book value decrease at increasing rate with respect to useful life.

Let, I = initial cost, S = salvage value, N = Life of the asset

i = Rate of return compounded annually

A = The annual equivalent amount of depreciation charge

B_N = Book value of the asset at the end of period n

D_n = Depreciation charge at the end of the period n

To find the annual equivalent amount (A) = (I – S) × (A/F, i%, N)

To Find the depreciation charge (D_n) = (I– S)×(A/F, i%, N)×(F/P,i%, n - 1)

To find the book value at the end of period t = I–(I– s) × (A/F, i%, N) × (F/P,i%, n)

Example: Compute the depreciation charge and book value in each year by using sinking fund method with following information.

Initial cost = Rs. 2,00,000

Salvage value = Rs. 20,000

Life of the asset = 6 years

i = 12%

Solution:

$$\text{Fixed annual depreciation (A)} = (I - S)\,(A/F, 12\%, 6)$$
$$= (200000 - 20000) \times \frac{0.12}{1.12^{6} - 1}$$
$$= 22180.63$$

Net depreciation for the year 1

$$d_1 = (I - S)\,(A/F, 12\%, 6) \times (F/P, 12\%, n - 1)$$
$$= 22180.63 \times 1 \quad (n = 1) = 22180.63$$

Net depreciation for the year 2

$$d_2 = (I - S)\,(A/F, 12\%, 6) \times (F/P, 12\%, 2 - 1)$$
$$= 22180.63 \times (1 + 0.12)^{1} = 24842.3$$

On continuing, we obtain the depreciation value & book value in following table

Year	Fixed depreciation (A)	Net depreciation (D_n)	Book value (B_n)
1	22180.63	22180.63	200000 – 22180.63 = 177819.37
2	22180.63	22180.63 × 1.2 = 24842.3	177819.37 – 24842.3 = 152977
3	22180.63	22180.63 × 1.2^2 = 27823.38	152977 – 27823.38 = 125153.69
4	22180.63	22180.63 × 1.2^3 = 31162.18	93991.51
5	22180.63	34901.65	59089.86
6	22180.63	39089.84	20000.02 ≈ (20000)(ok)

Determination of sinking fund in terms of Co-efficient of annual sinking fund:

The calculation of sinking fund depends upon the life of a building and also upon the rate of interest. When the life of a building is over, the owner can get back a certain amount on the sale of old building materials which is known as scrap value. This amount is considered as 10% of the building cost. Therefore, the calculation of sinking fund is made on 90% cost of the building.

Let S = Total amount of the sinking fund; I = Annual installment required;

i = Rate of interest expressed in decimal; n = number of years and

Ic = Co-efficient of annual sinking fund, so that I = Ic × S

The first annual installment would accumulate interest for (n – 1) years, the second for (n – 2) years and so on. Also the annual sinking fund for redemption of Rs. 1.00 would be Ic (as I = Ic × S and S = 1).

Consequently, the first installment would accumulate amount to $Ic(1 + i)^{n-1}$, the second to $Ic(1 + i)^{n-2}$ etc.

Hence $Ic\,[(1 + i)^{n-1} + (1 + i)^{n-2} + \ldots\ldots + (1 + i)^2 + (1 + i) + 1] = 1$

or, $Ic \dfrac{[(1 + i)^n - 1]}{(1 + i) - 1} = 1$

$$\therefore Ic = \frac{i}{(1 + i)^n - 1} \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots(1)$$

Equation (1) the co-efficient of sinking fund is an important expression frequently required for valuation.

Consequently, $I = Ic \times S = \dfrac{Si}{(1 + i)^n - 1} \ldots\ldots(2)$

Example: What annual sinking fund at 4.5% must be invested to produce Re. 1/- at the end of 20 years?

Solution:

Annual sinking fund may be expressed as $I = \dfrac{Si}{(1 + i)^n - 1}$

Where, I = Annual sinking fund required, S = Total amount of the sinking fund,

i = Rate of interest on sinking fund expressed in decimal, n = number of years.

Putting the respective values from the question in the above expression

$$I = \frac{1 \times 0.045}{(1 + 0.045)^{20} - 1} = \frac{1 \times 0.045}{(1.045)^{20} - 1} = \frac{0.045}{2.4117 - 1} = 0.0319$$

7.2.4 Sum of the year Digit method (SOYD)

It is also an accelerated depreciation method. In this method the annual depreciation rate for any year is calculated by dividing the number of years left (from the beginning of that year for which the depreciation is calculated) in the useful life of the asset by the sum of years over the useful life.

The depreciation amount in any year is calculated by multiplying the depreciation rate for that year with the total depreciation amount (i.e. difference between initial cost 'I' and salvage value 'S') over the useful life.

- Larger depreciation charges during the beginning years of asset and smaller depreciation charges as asset getting old.
- Depreciation charge is calculated from the ratio of the sum of the year's digit for the total useful life and remaining useful life at the beginning of particular year.

$$\text{SOYD deprecation} = \frac{\text{Remaining useful life at the beginning of particular year}}{\text{SOYD for the total useful life}} \times (I - S)$$

$$D_n = \frac{(N - n + 1)\,(I - S)}{SOYD}$$

SOYD = sum of years' digits over the useful life

$$= 1 + 2 + 3 + 4 + \ldots\ldots N = \frac{N(N+1)}{2}$$

Example: Compute the SOYD depreciation schedule for the following

Cost basis of the asset, I = Rs. 20,000

Useful life N = 5 years

Salvage value, SV = Rs, 2000

Solution: $SOYD = \frac{N(N+1)}{2} = \frac{5\,(5+1)}{2}$

$= 15$

Year	SOYD depression D_n	Book value B_N
1	$\frac{5}{15}(20000 - 2000) = 6000$	20000 – 6000 = 14000
2	$\frac{4}{15}(20000 - 2000) = 4800$	14000 – 4800 = 9200
3	$\frac{3}{15}(20000 - 2000) = 3600$	9200 – 3600 = 5600
4	$\frac{2}{15}(20000 - 2000) = 2400$	5600 – 2400 = 3200
5	$\frac{1}{15}(20000 - 2000) = 1200$	3200 – 1200 = 2000

7.2.5 Modified Accelerated Cost Recovery System (MACRS)

MACRS method allows taxpayers to deduct greater amounts during the first few years of an asset's life.

- Historically, for tax purpose and accounting an asset's depreciable life was determined by its estimated useful.
- The MACRS scheme, totally abandon this practice and simpler guidelines were set which created several classes of assets each with more or less arbitrary life called recovery period.
- The salvage value of property is always zero.
- MARCS scheme includes 8 categories of assets: 3 years, 5 year, 7 year, 10 year, 15 year, 20 year, 27.5 year and 39 year

Modified Accelerated Cost Recovery System (MACRS) Rules

Under earlier depreciation methods, the rate at which the value of an asset declined was estimated and was then used as the basis for tax depreciation. Thus, different assets were depreciated along different paths over time. The MACRS method, however, established prescribed depreciation rates, called recovery allowance percentages, for all assets within each class. These rates, as set forth in 1986 and 1993, are shown in Table below. The yearly recovery, or depreciation expense, is determined by multiplying the asset's depreciation base by the applicable recovery allowance percentage.

Half year convention

- It is assumed that all assets are placed at mid-year and they have zero salvage value.
- Half year depreciation is allowed for the first year and full year's depreciation is allowed in each of the remaining years of the assets recovery period and finally remaining half year depreciation in the end year of recovery period.
- With half of one year's depreciation being taken in the first year, a full year's depreciation is allowed in each of the remaining years of the asset's recovery period, and the remaining half-year's depreciation is taken in the year following the end of the recovery period. A half year of depreciation is also allowed for the year in which property is disposed of, or is otherwise retired from service, any time before the end of the recovery period.

- The MACRS asset is depreciated initially by the declining balance and then to straight line method.

MACRS Depreciation Rates

Year	3 year	5 year	7 year	10 year	15 year	20 year
1	33.33	20.00	14.29	10.00	5.00	3.75
2	44.45	32.00	24.49	18.00	9.50	7.219
3	14.81	19.20	17.49	14.40	8.55	6.667
4	7.41	11.52	12.49	11.52	7.70	6.177
5		11.52	8.93	9.22	6.93	5.75
6		5.76	8.92	7.37	6.23	5.285
7			8.93	6.55	5.90	4.888
8			4.46	6.56	5.90	4.522
9				6.55	5.91	4.462
10				6.55	5.90	4.461
11				3.28	5.91	4.462
12					5.90	4.461
13					5.91	4.462
14					5.90	4.461
15					5.91	4.462
16					2.95	4.461
17						4.462
18						4.461
19						4.462
20						4.461
21						2.231

Example: A tax payer wants to place in service a Rs. 20,000 asset that is assigned to the 5 year class. Compute the MACRS percentage depreciation amounts and book value for the asset.

Solution:

[Note: MACRs is obtain from given table but in exam table may not be given. Thus calculation is required which is shown below.]

MACRs deduction percentage, beginning with the first taxable year and ending with the 6th year.

Straight line rate = 1/5 = 0.2

Double declining balance rate $(\alpha) = \frac{1}{5} \times 200\% = 40\%$

Salvage value in MACRS = 0

Year	Calculation (%)	MACRs	Decision
1	$\frac{1}{2}$ year DDB dep = $0.5 \times 0.4 \times 100$	20%	
2	DDB dep = $0.4 \times (100 - 20)$	32%	Do not switch
	SL dep. = $\frac{1}{4.5} \times (100 - 20)$	17.78%	
3	DDB dep = $0.4 \times (100 - 20 - 32)$	19.2%	Do not switch
	SL dep. = $\frac{1}{3.5} \times (100 - 20 - 32)$	13.71%	
4	DDB dep = $0.4 \times (100 - 71.2)$	11.52%	Switch to SL
	SL dep. = $\frac{1}{2.5} \times (100 - 71.2)$	11.52%	
5	SL dep = $\frac{1}{1.5}(100 - 82.72)$	11.52%	
6	$\frac{1}{2}$ year SL dep = 0.5×11.52	5.76%	

In the year 4, SL depreciation is ≥ DDB depreciation and we switch to SL

<u>**Calculation the depreciation amounts from the percentages**</u>

Year	MARCS (%)	Depreciation (D_N)	Depreciation Amount (B_N)
1	20	$0.2 \times 20000 = 40000$	$20000 - 4000 = 16000$

2	32	0.32 × 20000 = 6400	16000 – 6400 = 9600
3	19.2	0.192 × 20000 = 3840	9600 – 3840 = 5760
4	11.52	0.1152 × 20000 = 2304	5760 – 2304 = 3456
5	11.52	0.1152 × 20000 = 2304	3456 – 2304 = 1152
6	5.76	0.0576 × 20000 = 1152	1152 – 1152 = 0

7.3 Introduction to Corporate Income Tax

Individuals as well as corporations are subjected to income tax. Income tax on corporation is known as the corporate income tax or corporate tax. Corporation is a word used to cover a variety of business enterprises, all of which are entities having a legal personality and distinct from their owners. There are various types of business enterprises but corporations and companies have only a separate legal personality. It is only a way of including all corporate source of income in the personal income tax base.

The corporate tax rate, is applied to the taxable income of a corporation.

The allowable deductions include the cost of goods sold, salaries and wages, rent, interest, advertising, depreciation, amortization, depletion and various tax payments other than federal income tax. The following table is illustrative:

Item
Gross income
Expenses:
Cost of goods sold
Depreciation
Operating expenses
Taxable operating income
Income taxes
Net income

Marginal tax rate: The amount of tax paid on an additional rupees of income.

Effective (average) tax rate: The rate a taxpayer would be taxed at if taxing was done at a constant rate, instead of progressively.

Example: A main-order computer company sells personal computers and peripherals. The company leased showroom space and a warehouse for $20,000 a year and installed $290,000 worth of inventory-checking and packaging equipment. The allowed depreciation expenses for this capital expenditure ($ 290,000) amounted to $58,000 using the category of 5-year MACRS. The store was completed and operations began on January 1. The company had a gross income of $1,250,000 for the calendar year. Supplies and all operating expenses, other than the lease expense, were itemized as follows:

Merchandise sold in the year	$600,000
Employee salaries and benefits	150,000
Other supplies and expenses	90,000
	$840,000

Complete the taxable income for this company. How much will the company pay in federal income taxes for the year?

Solution:

Given: Income, preceding cost information and depreciation.

Find: Taxable income and federal income taxes.

First we compute the taxable income as follows:

Gross revenues	$1,250,000
Expenses	-840,000
Lease expenses	-20,000
Depreciation	-58,000
Taxable income	$332,000

Note: the capital expenditures are not deductible expenses. Since the company is in the 39% marginal tax bracket, its income tax can be calculated by using the formula given in Table 9.6, 22,250 + 0.39 (X - 100,000):

Income tax = \$22,250 + 0.39(\$332,000 – \$ 100,000) = \$ 112,730

The firm's current marginal tax rate is 39%, but its average corporate tax rate is $\frac{\$112,730}{\$ 332,000}$ = 33.95%

7.4 After Tax Cash Flow Estimate

It is measure of a company's ability to generate positive cash flow after deducting taxes.

The general formula for after tax cash flow is

ATCF = Net income + Depreciation + Amortization

Sometimes analysis also add back other non-cash items and proceeds from debt or equity issuance

The amount remaining after expenses, mortgage payments, and income taxes have been deducted from the gross income of an investment property is the after tax cash flow.

7.5 General Procedure for Making after Tax Economic Analysis

7.5.1 Some terms and Definition

Gross Income (GI) – total income realized from all revenue-producing sources, including items such as the sales of assets, royalties, license fees etc.

Income Tax – amount of taxes based on gross income. Corporate taxes are typically paid quarterly, and are actual cash flows.

Operating Expenses (*E*) – all corporate costs incurred in the transaction of business.

Taxable Income (TI) – the amount upon which taxes are based.

Tax Rate (*T*) – percentage of TI owed in taxes. This rate is graduated based on TI as per country.

Net Profit after taxes (NPAT) – amount remaining each year when income taxes are subtracted from taxable income.

Average Tax Rate – because the marginal tax rate varies as TI varies, the average tax rate is calculate as:

Avg. tax rate = total taxes / TI

Effective Tax Rate (T_e) – the total rate paid by corporations, including federal, state and local taxes. Note state taxes can be deducted from federal taxes. So:

T_e = state rate + (1-state rate)(federal rate)

CFBT Vs CFAT

Cash flow before tax (CFBT) – all cash flows throughout the year without considering taxes. Note, all our PW, FW, AW analysis to this point have been CBFT cash flows.

CFBT = GI – *E* – *P* + *S*, Where *P* is initial investments and *S* is salvage.

Cash flow after tax (CFAT) – includes the cash flow impact of taxes.

CFAT = CFBT – taxes

- Taxes are calculated taking depreciation (*D*) into account, however depreciation is not a cash flow, but taxes are.

Taxes = TI * (T_e)

TI = GI – *E* – *D*

CFAT = GI – E – P + S – (GI – E – D) * (T_e)

Capital Gains (CG): Occurs when selling price is greater than first cost.

Capital gain = selling price – first cost

CG = SP – P

Depreciation Recovery (DR): Occurs when a depreciable asset is sold for more than the current book value.

Depreciation recapture = selling price – book value

DR = SP – BV

Capital Loss (CL): Occurs when a depreciable asset is disposed of for less than its current book value.

CL = BV_t – SP

Considering capital gains, depreciation recovery and Capital losses,

TI = gross income – expenses – depreciation + depreciation recapture + capital gains – capital loss

TI = GI – E – D + DR + CG – CL

7.5.2 General Procedure for ATCF

Let, R_k = revenues from the project during period k

E_k = Expenses during year k

d_k = depreciation and depletion during year k

t = Effective income tax rate

T_k = income tax consequences during year k

$ATCF_k$ (After tax cash flow) = ATCF from the project during year k

Because the NIBT (i.e. taxable income) is $(R_k - E_k - d_k)$, the ordinary income tax consequences during year k are computed with

$$T_k = -t\,(R_k - E_k - d_k)$$

Therefore, when $R_k > (E_k + d_k)$, a tax liability occurs when $R_k < (E_k + d_k)$, a decrease in the tax amount occurs.

$NIAT_k$ (Net income after Tax) = $\underbrace{(R_k - E_k - d_k)}_{\text{Taxable income}} - \underbrace{t\,(R_k - E_k - d_k)}_{\text{Income taxes}}$

$$NIAT_k = (R_k - E_k - d_k)(1 - t)$$

The ATCF associated with a project equals the NIAT plus non-cash items such as depreciation.

$$ATCF_k = NIAT_k + d_k$$

$$= (R_k - E_k - d_k)(1 - t) + d_k$$

$$ATCF_k = (1 - t)(R_k - E_k) + td_k$$

Example: If the revenue from a project is $10,000 during a tax year, out-of-pocket expenses are $4000 and depreciation deductions for income tax purposes are $ 2,000 what is the ATCF when t = 0.4? What is the NIAT?

Solution:

$$ATCF = (1 - t)(R_k - E_k) + td_k$$

$$= (1 - 0.4)(10000 - 4000) + 0.4 \times 2000 = \$\,4{,}400$$

Also,

$$ATCF_k = NIAT_k + d_k$$

$$NIAT = ATCF - d_k = 4400 - 2000 = \$\,2400$$

Old Question Solutions

1. Define depreciation and list out important methods of calculating depreciation deductions. **(2069 Bhadra)**

Solution:

Definition of depreciation: Refer at 7.1

Important methods of calculating depreciation

1. Straight line method
2. Declining balance method
3. Sinking fund method
4. SOYD method
5. Modified accelerate cost recovery system (MACRs)
6. Service output method

2. A machine costs Rs. 15000. Its useful life is 5 years and salvage value is Rs. 900. Compute the annual depreciation allowance and resulting book values using double declining balance depreciation methods. **(2069 Bhadra)**

Solution:

Rate of depreciation = $\frac{1}{N} \times 2 = 0.4 = 40\%$

End of year	Depreciation (DB)	SL Dep.	Decision	Book value
1	$0.4 \times 15000 = 6000$	$\frac{(15000 - 900)}{5} = 2820$	Not switch	$15000 - 6000 = 9000$
2	$0.4 \times 9000 = 3600$	$\frac{(9000 - 900)}{4} = 2025$	Not switch	$9000 - 3600 = 5400$
3	$0.4 \times 5400 = 2160$	$\frac{(5400 - 900)}{3} = 1500$	Not switch	$5400 - 2160 = 3240$
4	$0.4 \times 3240 = 1296$	$\frac{(3240 - 900)}{2} = 1170$	Not switch	$3240 - 1296 = 1944$
5	$0.4 \times 1944 = 777.6$	$\frac{(1944 - 900)}{1} = 1044$	Switch	$1944 - 1044 = 900$

3. Define depreciation. What are the causes for it? It a machine costing of Rs. 1,50,000 is purchased by expecting salvage value Rs.

40,000 at the end of 6th year. Calculate the depreciation amount for each year by,

i) SOYD

ii) Declining balance (2070 Magh)

Solution:

First part Refer at 6.1

$$SOYD = \frac{N(N+1)}{2}$$

$$= \frac{6 \times 7}{2} = 21$$

Year	Depreciation (D_N)	Book value (B_N)
1	$\frac{6}{21}$ (1,50,000 – 40,000) = 31428.57	150000 – 31428.57 = 118571.43
2	$\frac{5}{21}$ (1,50,000 – 40,000) = 26190.47	118571.43 – 26190.47 = 92380.96
3	$\frac{4}{21}$ (1,50,000 – 40,000) = 20952.38	92380.96 – 20952.38 = 71428.58
4	$\frac{3}{21}$ (1,50,000 – 40,000) = 15714.28	71428.58 – 15714.28 = 55714.3
5	$\frac{2}{21}$ (1,50,000 – 40,000) = 10476.19	55714.3 – 10476.19 = 45238.09
6	$\frac{1}{21}$ (1,50,000 – 40,000) = 5238.095	45238.09 – 5238.09 = 40000

ii) Declining balance

Depreciation rate,

$$(R) = 1 - \left(\frac{Sv}{p}\right)^{\frac{1}{N}}$$

$$= 1 - \left(\frac{40000}{15000}\right)^{1/6}$$

$$= 0.1977$$

$$= 19.77\%$$

Year	B_{N-1}	D_N by D_B method	B_N
1.	1,50,000	1,50,000 × 0.9177 = 29655	1,20,315
2	1,20,345	1,20,345×0.1977 = 23,892	96,553
3	96,553	19,088	77,465
4	77,465	15315	62,150
5	62,150	12,287	49863
6	49,863	9,858	40,005

4. Suppose an equipment purchased for Rs. 10,00,000. It is expected to generate income of Rs. 3,50,000 per year during 5 years and corporate income tax rate is 25% per year. Under recovery periods depreciation are as follows.

Year	1	2	3	4	5
Depreciation amount	1,00,000	2,00,000	2,00,000	2,00,000	1,00,000

Calculate ATCFs and determine profitability (IRR) when MARR is 15% by using PW method. (2070 Magh)

Solution:

Year	Cash flow	Depreciation	Taxable income (Cash flow - Depreciation)	Tax = 0.25 × Taxable income	ATCF = Cash flow - tax
0	- 10,00,000	-	-	-	- 10,00,000
1	3,50,000	1,00,000	2,50,000	62500	287500
2	3,50,000	2,00,000	1,50,000	37500	312500
3	3,50,000	2,00,000	1,50,000	37500	312500
4	3,50,000	2,00,000	1,50,000	37500	312500
5	3,50,000	1,00,000	2,50,000	62500	287500

Calculation of profitability (IRR) ATCF

$$PW(i\%) = -10,00,000 + \frac{287500}{(1+i)} + \frac{31250}{(1+i)^2} + \frac{312500}{(1+i)^3} + \frac{312500}{(1+i)^4} + \frac{287500}{(1+i)^5} = 0$$

On solving, we get

i = 15.56% > MARR (15%)

Economically justified.

5. **An asset has installed value of 45,000. S_v = 0. It is classed as a 5 year property. Determine approximate MACRS depreciation schedule. Over 6 years it is estimated to generate revenue of Rs. 23,000 per year with annual operating cost 7300. Required rate of return = 15% after tax. Tax rate = 40%. Evaluate after tax IRR with annual worth method.**

(2070 Bhadra)

Solution:

Installed Value = 45,000 Revenue = 23,000/year. O & M = 7300/year

Year	MACRs % calculation	MACRs	Depreciation amount	BTCF	Taxable income	Tax 0.4 × TI	ATCF (BTCF - tax)
0.	-	Switching	45000 × MACRs	- 45000	-	-	- 45000
1. (half year)	DB = $\frac{1}{2}$ × 0.2 ×100 = 20%	20%	0.2 × 45000 = 9000	$\frac{1}{2}$ × (23000 - 7300) = 7850	7850 - 9000 = - 1150	-	7850
	SL = $\frac{1}{2}\times\frac{1}{5}$ × 100 = 10%	(Do not switch)					
2.	DB = 0.4 × (100 - 80) = 32%	32%	0.32 × 45000 = 14400	(23000 - 7300) = 15700	15700 - 14400 = 1300	0.4 × 1300 = 520	15700 - 520 = 15180
	SL = $\frac{1}{4.5}$ × 80 = 17.78%	(Do not switch)					
3.	DB = 0.4 × 48 = 19.2	19.2%	0.192 × 45000 = 8640	(23000 - 7300) = 15700	15700 - 8640 = 7060	0.4 × 7060 = 2824	15700 - 2824 = 12876
	SL = $\frac{1}{3.5}$ × 48 = 13.71	(Do not switch)					
4.	DB = 0.4 × 28.8 = 11.52	11.52	0.1152 × 45000 = 5184	(23000 - 7300) = 15700	15700 - 5184 = 10516	0.4 × 10516 = 4206.4	15700 - 4206.4 = 11493.6
	Sl = $\frac{28.8}{2.5}$ = 11.52	Switch					
5.	SL = $\frac{17.28}{1.5}$ = 11.52	11.52	0.1152 × 45000 = 5184	(23000 - 7300) = 15700	15700 - 5184 = 10516	0.4 × 10516 = 4206.4	15700 - 4206.4 = 11493.6
6	Half year dep = $\frac{1}{2}$ × 11.25 = 5.76	5.76	0.0576 × 45000 = 2592	$\frac{1}{2}$ × (23000 - 7300) = 7850	7850 - 2592 = 5258	0.4 × 5258 = 2103.2	7850 - 2103.2 = 5746.8

Where BTCF = Before Tax cash flow

ATCF = After Tax cash flow

[Note: If taxable income is negative, tax can be neglected]

Also,

$$\text{PW of ATCF (15\%)} = -45000 + \frac{7850}{1.15} + \frac{15180}{1.15^2} + \frac{12876}{1.15^3} + \frac{11493.6}{1.15^4} + \frac{11493.6}{1.15^5} + \frac{5746.8}{1.15^6}$$

$$= -3459.22\ (< 0)$$

Economically not justified.

(Note: Two table can be prepare to make calculation easy)

6. **Compute the book value at the end of 3 years (BV3) by all the methods of depreciation except MACRS method. Cost basis of a machine is Rs. 10,000. SV = 0. Useful life = 5 years. MARR = 10%**

(2071 Magh)

Solution:

I) **Straight line method**

$$\text{Depreciation} = \frac{10000 - 0}{5} = 2000 \text{ per year}$$

Book value at the end of 3 year = 10000 - 3 × 2000= 4000

II) **Declining balance method**

$$\text{rate of depreciation } (\alpha) = \frac{1}{5} \times 2 \times 100 = 40\%$$

$$\text{Book value at the end of 3 year} = 1000(1-\alpha)^3 = 10000(1-0.4)^3 = 2160$$

III) Sinking fund method

Book value at the end of 3 year

$I-(I-S)\times(A/F,i\%,N)\times(F/A,i\%,t)$

$= 10000-(10000-0)\dfrac{0.1}{1.1^5-1}\times\dfrac{1.1^3-1}{0.1}$ $\left[\begin{matrix} t=3 \\ N=5, i=10\end{matrix}\right.$

$= 4578.3$

IV) SOYD

SOYD $= \dfrac{5(5+1)}{2} = 15$ (For N = 3)

Sum of year $= 1+2+3 = 6$ (For n = 3)

Book value at the end of 3 year $= 10000-\dfrac{6}{15}\times(10000-0) = 6000$

7. Evaluate after tax PW. The cost basis for a machine is Rs. 10,000 the machine is 5 years MACRs property over 6 years, it is estimated to save Rs. 4500 per year in maintenance costs with annual operating cost being Rs. 1000. It will be depreciated by MACRS method. SV = 0 Tax rate = 30%, MARR = 15%. (2071 Magh)

Solution:

Total cost (I) = 10000

Revenue = 4500, O & M = 1000

Year	MACRs (Same as 2070 Bhadra)	Depreciation (Dn) amount 10000 × MACRS	BTCF	Taxable income BTCF − D_n	Tax Taxable income × 0.3	ATCF (BTCF − Ta…
0	-	-	− 10000	-	-	−1000…
1	20%	0.2 × 10000 = 2000	$\frac{1}{2}\times(4500-1000) = 1750$	(1750 − 2000) = −250	-	1750−… =175…
2	32%	0.32 × 10000 = 3200	4500 − 1000 = 3500	3500 − 3200 = 300	0.3 × 300 = 90	3500−9… =341…
3	19.2%	1920	45000 − 1000 = 3500	3500 − 1920 = 1580	0.3 × 1580 = 474	3500−47… =302…
4	11.52%	1152	3500	3500 − 1152 = 2348	0.3 × 2348 = 704.4	3500−704… =2795…
5	11.52%	1152	3500	3500 − 1152 = 2348	0.3 × 2348 = 704.4	3500−704… =2795…
6	5.676%	576	$\frac{1}{2}(4500-1000) = 1750$	1750 − 576 = 1174	0.3 × 1174 = 352.2	1750−352… =1397…

$PW = -1000+\dfrac{1750}{1.15}+\dfrac{3410}{1.15^2}+\dfrac{3026}{1.15^3}+\dfrac{2795.6}{1.15^4}+\dfrac{2795.6}{1.15^5}+\dfrac{1397.8}{1.15^6} = -317.55 < 0$

Economically not justified.

8. What do you mean by depreciation? Explain about the causes of it. Explain about any three methods of deprecation calculation that are used commonly. A machine purchased for Rs. 60000 by expecting useful life of 10 years. Calculate the depreciation amount for each year by using deciding balance method when rate of depreciation is 20% per year. (2071 Bhadra)

Solution:

Theory part, Refer at 7.1 and 7.2

Investment = 60000, life = 10

depreciation rate = 20% per year

Salvage value is not given, so assume book value at the end of 10 year is exactly equal to the salvage value, (but its value is not less than 2000)

$\alpha = \frac{1}{N}\times 2 = 20\%$ per year = Double deciding balance rate

Year	B_{N-1}	Depreciation D_n	Book value B_N
1	60000	0.2 × 60000 = 12000	48000 (60000 − 12000)
2	48000	0.2 × 48000 = 9600	38400
3	38400	7680	30720
4	30720	6144	24576
5	24576	4915.3	19660.8
6	19660.8	3932.1	15728.64
7	15728.64	3145.8	12582.9
8	1258.29	2516.5	10066.3
9	10066.3	2013.2	8053.1
10	8053.1	1610.62	6442.48 (>0) ok

Salvage value at the end of 10 year = 6442.48

9. Explain the general procedure for after tax economic analysis with suitable example. (2072 Ashwin)

Solution: *Refer at 7.5*

10. Considering the following information, compute the [illegible] depreciation and book value of each year by. (i) SL method (ii) [illegible] method (iii) SOYD method and (iv) sinking fund method

Cost basis	Salvage value	Useful life	MARR
$ 7000	$ 2000	5 years	10%

(2072 Ashwin)

Solution:

i) SL method

$$Dn = \frac{(7000 - 2000)}{5} = 1000$$

N	B_{N-1}	D_n	B_N
1	7000	1000	6000
2	6000	1000	5000
3	5000	1000	4000
4	4000	1000	3000
5	3000	1000	2000

ii) DB method

$$\alpha = \frac{1}{5} \times 2 = 0.4 = 40\%$$

N	B_{N-1}	D_n	Decision	B_N
1	7000	0.4 × 7000 = 2800		4200
2	4200	0.4 × 4200 = 1680		2520
3	2520	1008	Dn > (2520 – 2000 = 520)	2000
4	2000	0		2000
5	2000	0		2000

iii) SOYD method

$$SOYD = \frac{5(5+1)}{2} = 15$$

N	B_{N-1}	D_n	B_N
1	7000	$\frac{5}{15}(7000 - 2000) = 1666.67$	5333.33
2	5333.33	$\frac{4}{15}(7000 - 2000) = 1333.33$	4000
3	4000	$\frac{3}{15}(7000 - 2000) = 1000$	3000
4	3000	$\frac{2}{15}(7000 - 2000) = 666.67$	2333.33
5	2333.33	$\frac{1}{15}(7000 - 2000) = 333.33$	2000 (OK)

iv) Sinking Fund method

Fixed annual depreciation (A) $= (I - S)(A/F, 10\%, 5)$

$$= (7000 - 2000) \times \frac{0.1}{1.1^5 - 1} = 818.98$$

Net depreciation for the year 1

$= (I - S)(A/F, 10\%, 5) \times (F/P, 10\%, 1 - 1) = 818.98 \times 1 = 818.98$

Depreciation for the year 2

$= 818.98 \times 1.1 = 900.88$

Similar for others, in table

N	B_{N-1}	D_n	B_N
1	7000	$818.98 \times 1 = 818.98$	6181.02
2	6181.02	$818.98 \times 1.1 = 900.88$	5280.14
3	5280.14	$818.98 \times 1.1^2 = 990.96$	4289.18
4	4289.18	$818.98 \times 1.1^3 = 1090.06$	3199.12
5	3199.12	$818.98 \times 1.1^4 = 1199.06$	$2000.06 \approx 2000$

11. Write down the causes for depreciation of assets. If a machine casting of Rs. 1,00,000 is purchased by expecting salvage valve of Rs. 20,000 at the end of 6th years. Calculate the depreciation amount for each year by SOYD and straight line method.

(2073 Bhadra)

Solution:

Causes of depreciation of assets

1) Wear and Tear
2) Effusion of time
3) Obsolescence
4) Exhaustion or depletion
5) Accident
6) Fall in market value

Second Part

Given, Investment = 1,00,000

Salvage value = 20,000

SOYD method

$$SOYD = \frac{N(N+1)}{2} = \frac{6*(6+1)}{2} = 21$$

Year	Depreciation (D_N)	Book Value (B_N)
1	$\frac{6}{21}$ (1,00,00 – 20,000) = 22857.17	1,00,000 – 22857.14 = 77142.86
2	$\frac{5}{21}$ (1,00,000 – 20,000) = 19047.62	77142.86 – 19047.62 = 58095.24
3.	$\frac{4}{21}$ (1,00,000 – 20,000) = 15238.09	58095.24 – 15238.09 = 42857.15
4.	$\frac{3}{21}$ (1,00,000 – 20,000) = 11428.57	42857.15 – 11428.57 = 31428.58
5.	$\frac{2}{21}$ (1,00,000 – 20,000) = 7619.04	31428.58 – 7619.04 = 23809.54
6.	$\frac{1}{21}$ (1,00,000 – 20,000) = 3809.54	23809.54 – 3809.54 = 20000

SL Method

$$D_n = \frac{100,000 - 20,000}{6}\left(\frac{I-S}{N}\right) = 13333.33$$

Year	B_{N-1}	D_n	B_N
1	1,00,000	13333.33	1,00,000 – 13333.33 = 86666.67
2	86666.67	13333.33	73333.34
3	73333.34	13333.33	60000
4	60000	13333.33	46666.68
5	46666.68	13333.33	333333.35
6	33333.37	13333.33	≈20000

12. If a machine has following information, compute the depreciation and book value of each year by

i) SOYD method ii) Sinking fund method **(2073 Magh)**

Cost basis	Salvage value	Life	MARR
$ 8000	$ 2000	10 years.	10%

Solution:

i) SOYD method

$$SOYD = \frac{N(N+1)}{1} = \frac{10 \times 11}{2} = 55$$

N	B_{N-1}	D_n	B_N
1	8000	$\frac{10}{55}$ (8000 – 2000) = 1090.9	8000 – 1090.9 = 6909.1
2	6909.1	$\frac{9}{55}$ (8000 – 2000) = 981.82	6901.1 – 981.82 = 5927.23
3	5927.28	$\frac{8}{55}$ (8000– 2000) = 872.73	5054.55
4	5054.55	763.63	4290.92
5	4290.92	654.54	3636.38
6	3636.38	545.45	3090.93
7	3090.93	436.36	2654.57
8	2654.57	327.27	2327.3
9	2327.3	218.18	2109.12
10	2109.12	109	2000.12 = 2000

ii) **Sinking find method**

Fixed annual depreciation (A) = (I – S) (A/F, 10%, 10)

$$= (8000 - 2000) * \frac{0.1}{1.1^{10} - 1}$$

= 376.47

Net depreciation for the year 1

= (I – S) (A/F, 10%, 10) * (F/P, 10%, 1 – 1)

= 376.47 × 1 = 376.47

Depreciation for the year 2

= 376.47 *1.1 =414.12

Similar for others, in table

N	B_{N-1}	D_n	B_N
1	8000	376.47 × 1 = 376.47	8000 − 376.47 = 7623.53
2	7623.53	376.47 × 1.1 = 414.117	7623.53 − 414.117 = 7209.4
3	7209.4	$376.47 \times 1.1^2 = 455.53$	6753.88
4	6753.88	$376.47 \times 1.1^3 = 501.08$	6252.8
5	6252.8	$376.47 \times 1.1^4 = 551.18$	5701.62
6	5701.62	$376.47 \times 1.1^5 = 606.308$	5095.3
7	5095.3	$376.47 \times 1.1^6 = 666.93$	4428.38
8	4428.38	$376.47 \times 1.1^7 = 733.63$	3694.755
9	3694.755	$376.47 \times 1.1^8 = 806.99$	2887.76
10	2887.76	$376.47 \times 1.1^9 = 887.69$	2000.075 ≈ 2000

13. Explain about the method of "MACRS" depreciation. (2073 Magh)

Solution:

Refer 7.2.5

14. Compute the annual depreciation allowances and the resulting book value using the double declining balance method with switch over straight line method. Cost of asset = Rs. 100, 000, useful life = 5 yrs salvage value = 20000. (2074 Bhadra)

Solution:

From Given data

$$I = 100{,}000$$
$$N = 5 \text{ yrs}$$
$$S = 20{,}000$$

Now, decling depreciation rate (α) = $\frac{1}{N} \times 2 = \frac{1}{5} \times 2 = 40\,\% = 0.4$

E OY	Decline dep.(DN)	SL dep	Decision	book value(BN)
1	0.4 ×100000 = 40000	$\frac{100000 - 200000}{5}$ = 16000	No switch	100000 − 40000
2.	0.4 × 60000 = 24000	$\frac{60000 - 20000}{4}$ = 10000	No switch	60000 − 24000 = 36000
3.	0.4×36000 = 14400	$\frac{36000 - 20000}{3}$ = 5333.33	No switch	36000 − 14400 = 21600
4.	0.4 × 21600 = 8640	$\frac{21600 - 20000}{2}$ = 800	sv = 200000 21600−20000 = 1600 < DB switch to SL	21600 − 1600 = 20000 (i.e $B_N = S$)
5.	0	0	-	20000

[Note: If decline Depreciation is less than SL depreciation "switch" otherwise "No switch".

If BN < s, then depreciation amounts are adjusted so that BN = S by switching.]

15. A company bought a machine at Rs. 25000 which is expected to produce benefit of Rs. 8000 per year for five years. It's salvage at the end of five years is Rs. 1000 calculate after tax cash flow if Tax rate is 40% and depreciation is on sinking found method , I = 20%.

(2074 Bhadra)

Solution:

Investment (I) = Rs: 25000

Annual Benefit = Rs. 8000

N = 5 years

Salvage value (s) = Rs. 10000

Tax rate = 40% per year

I = 20%

First calculating depreciation by sinking fund method

Fixed Annual depreciation (A) = (I − S) (A/F, 20%, 5)

$$= (25000 - 10000) \times \frac{0.2}{1.2^5 - 1}$$

$$= 2015.70$$

Net depreciation charge for the year 1

d_1 = (I − s) (A/F, 20%, 5) × (F/P, 20%, n − 1)

= 2015.70 × 1 (∴ n = 1)

= 2015.90

For year 2 (i. e n = 2)

= (I − S) (A/f, 20%, 5)_ × (F/p, 20% n −1)

$= 2015.70 \times (1 = 0.2)^1$

$= 2418.83$

Similarly for your 1 to 5 in tabular form.

year	fixed Annual depreciation	Net depreciation
1.	2015.70	2015.70
2.	2015.70	2418.83
3.	2015.70	2902.6
4.	2015.70	3483.13
5.	2015.70	4179.75

Again for after tax cash flow

Year	cash flow	Depreciation	Taxable income (cash five-depreciation)	Tax = 0.4 × taxable income	ATCF = cash flow - Tax
0	– 25000	–	–	–	–25000
1	8000	2015.70	5984.3	2393.72	5606.28
2.	8000	2418.83	5581.16	2232/464	5767.54
3.	8000	2902.6	5097.39	2038.95	5961.00
4.	8000	3483.19	4516.87	1806.75	6193.25
5.	8000	4179.75	3820.24	1528.10	6471.90

16. Define depreciation. What are the advantages of depreciation concept? Your college is considering purchase vehicle of Rs 4,00,000 that assigned 5 years useful life and expected salvage value is Rs 1,00,000. Calculate depreciation for each year by using declining balance and MACRS. (2075 Bhadra)

Solution:

For definition port [Refer 7.1]

Given I = 400000

S = 100000

N = 5 year

Declining balance rate (α) $= \frac{1}{N} \times 2 = \frac{1}{5} \times 2 = 40\ \% = 0.4$

By using double declining balance method.

Year (N)	B_{N-1}	Depreciation ($D_N = \alpha * B_{N-1}$)	$B_N = B_{N-1} - D_N$
1.	400000	160000	240000
2.	240000	96000	144000
3.	144000	57600	86400
4.	86400	34560	51840
5.	51840	20736	31104

Note: Calculator Shortcut trick

Assume $B_{N-1} = A, D_N = B, B_N = C$

Then write the equation in the form of

$A: B = 0.4 \times A : C = A - B : A = C$

Then Press (CALC)

At first input, give A = 400000 then Press (=), it automatically give the value of B (i.e D_N) and C (i.e. B_N) and value of A will automatically replace by new value of C.

Again,

By using MACRS method:

[Note: MACRs is obtain from given table but in exam table may not be given. Thus calculation is required which is shown below.]

MACRs deduction percentage, beginning with the first taxable year and ending with the 6th year.

Straight line rate = 1/5 = 0.2

Double declining balance rate (α) $= \frac{1}{5} \times 200\% = 40\%$

Salvage value in MACRS = 0

Year	Calculation (%)	MACRs	Decision
1	$\frac{1}{2}$ year DDB dep $= 0.5 \times 0.4 \times 100$	20%	
2	DDB dep $= 0.4 \times (100 - 20)$	32%	Do not switch

	SL dep. $= \frac{1}{4.5} \times (100 - 20)$	17.78%	
3	DDB dep $= 0.4 \times (100 - 20 - 32)$	19.2%	Do not switch
	SL dep. $= \frac{1}{3.5} \times (100 - 20 - 32)$	13.71%	
4	DDB dep $= 0.4 \times (100 - 71.2)$	11.52%	Switch to SL
	SL dep. $= \frac{1}{2.5} \times (100 - 71.2)$	11.52%	
5	SL dep $= \frac{1}{1.5} (100 - 82.72)$	11.52%	
6	$\frac{1}{2}$ year SL dep $= 0.5 \times 11.52$	5.76%	

In the year 4, SL depreciation is ≥ DDB depreciation and we switch to SL.
Now, calculating the depreciation amounts from the percentages.

Year	MACRS(%)	Depreciation(DN)	BOOK Value(B_N)
1.	20	$0.2 \times 4000000 = 80000$	$400000 - 800000 = 320000$
2.	32	$0.32 \times 4000000 = 128000$	$320000 - 118000 = 192000$
3.	19.2	$0.192 \times 4000000 = 76800$	$192000 - 76800 = 115200$
4.	11.52	$0.115 \times 4000000 = 46080$	$115200 - 46080 = 69120$
5.	11.52	46080	69,20 − 46080 = 23040
6.	5.76	$0.0876 \times 4000000 = 23040$	$23040 - 23040 = 0$

17. What do you mean by tax, personal tax and corporate tax? Develop a model to calculate after tax cash flow. **(2075 Bhadra)**

Solution: *Refer [chapter 7.3] for definition.*

Refer [Chapter 7.5] for after tax cash flow analysis.

18. Explain the general procedure for after tax economic analysis with suitable examples. **(2076 Bhadra)**

Solution: *Refer [chapter 7.5]*

19. Compute annual depreciation and book value of year using following depreciation methods: (i) Declining Balance Method (ii) SOYD Method (iii) Sinking Fund Method **(2076 Bhadra)**

Cost Basis	Salvage Value	Useful Life	MARR
$ 7000	$ 2000	5 Years	10%

Solution: *Refer solution of 2073 Magh.*

Extra Question Solutions

1. A machine costing of Rs. 4000 is estimated to have life of 10 years. Find

i) Depreciation amount for the 6th year

ii) Accumulated depreciation throughout 6th year

iii) Book value at the end of 6th year

If Rate of depreciation = 20% and there is no salvage value by using Matheson method.

Solution:

i) Depreciation amount for the 6th year

$$d_6 = \alpha I (1 - \alpha)^{n-1}$$
$$= 0.2 \times 4000 (1 - 0.2)^{6-1}$$
$$= \text{Rs. } 262.144$$

ii) Accumulated depreciation throughout 6th year =

$$D_6 = I [1 - (1 - \alpha)^n]$$
$$= 4000 [1 - (1 - 0.26)^6]$$
$$= \text{Rs. } 2951.42$$

iii) Book value at the end of 6th year,

$$Bv_6 = I (1 - \alpha)^n$$
$$= 4000 (1 - 0.2)^6$$
$$= \text{Rs. } 1048.576$$

2. Say you have a property with ten tenants each paying Rs. 10000 per month. You estimate a vacancy loss of 7%. The property has operating expenses of Rs. 45000 per year and a first mortgage payment of Rs. 36,326 per year. In month six, you add a new roof at the cost of Rs. 20000 and take out Rs. 20000 second mortgage to cover the cost of that construction. Your payment on this loan totals Rs. 881 for the remaining six months. What is your property's cash flow before tax (CFBT)? If your tax liability in year one is 5000, then what is your property's cash flow after taxes (CFAT)?

Solution:

Gross Schedule income = 1000 per month = 120000 per month

Vacancy loss = $0.07 \times 12000 = 8400$

Gross operating income = 120000 - 8400 = 111600

Operating expenses = 45000

Gross operating income - operating expenses = 111600 - 45000 = 66600

∴ Net operating income = Rs. 66600

Cash flow before tax (CFBT) = Net operating income - debt service - capital addition + loan proceed

= 66600 - 36326 - 20000 + 20000

= Rs. 30274

Cash flow after tax (CFAT) = CFBT - tax liability

= 30274 - 5000

= Rs. 25274

***

8 INFLATION & ITS IMPACT ON PROJECT CASH FLOWS

8.1 Concept of Inflation

Inflation is the rate at which the general level of prices, good and services is rising, and subsequently purchasing power is falling.

Historically, the general economy has usually fluctuated in such a way as to exhibit inflation, a loss in the purchasing power of money over time. Inflation means that the cost of an item tends to increase over time or dollar amount buys less of an item over time.

An increase in demand and decrease in supply are two causes of inflation. Inflation can simply be categorized as:

i) Demand pill inflation

ii) Cost pull inflation.

Deflation: Deflation is the opposite of inflation, is that prices usually decreases over time, hence a specific dollar amount gains in purchasing power.

8.2 Measuring Inflation

Economists have developed a measure called the **Consumer Price Index (CPI)**, which is based on a typical **market basket** of goods and services required by the average consumer. This market basket normally consists of items from eight major groups:

(1) Food and alcoholic beverages, (2) housing, (3) apparel, (4) transportation, (5) medical care, (6) entertainment, (7) personal care, and (8) other goods and services.

The consumer price index is a good measure of the general increase in prices of consumer products. However, it is not a good measure of industrial price increases. In performing engineering economic analysis, the appropriate price indexes must be selected to estimate the price increases of raw materials, finished products, and operating costs

Generally, inflation is measured in percentage term which is simply obtained by calculating the percentage change in current price index over the previous year. Price indices are developed on the basis of market prices of various goods and services under the study and it is used for measuring inflation.

Average inflation: To account for the effect of varying yearly inflation rates over a period of several years, we can compute a single rate that represents an average inflation rate.

Since each individual year's inflation rate is based on the previous year's rate, all these rates have a compounding effect. As an example, suppose we want to calculate the average inflation rate for a two year period: The first year's inflation rate is 4%, and the second year's rate is 8%, on a base price of $100.

Step 1. To find the price at the end of the second year, we use the process of compounding:

$\$100(1 + 0.04)(1 + 0.08) = \$112.32$

Step 2. To find the average inflation rate *f*, we establish the following equivalence equation:

$\$100(1 + f)^2 = \$112.32$ or $\$100(F/P, f, 2) = \$112.32$.

Solving for *f* yields, $f = 5.98\%$.

We can say that the price increases in the last two years are equivalent to an average annual percentage rate of 5.98% per year. Note that the average is a geometric, not an arithmetic, average over a several-year period. Our computations are simplified by using a single average rate such as this, rather than a different rate for each year's cash flows.

The most commonly used indices are:

1. **Consumer price index (CPI):** CPI is an inflationary indicator that measures the change in the cost of a fixed basket of products and services.
2. **Wholesale price index (WPI):** WPI is the price of representative basket of wholesale goods. It measures and tracks the changes in the price of goods in the stages before the retail level.

3. **GDP deflator:** GDP is a measure of the level of prices of all new, domestically produced, final goods and services in an economy. GDP stands for gross domestic product, the total value of all final goods and services produced within that economy during a specified period.
4. **Producer Price Index (PPI):** The PPI is good measure of the industrial price increase.

Example: Calculate the average inflation rate for a 2-years period. The first year's inflation rate is 5% and the second year's rate is 10% using a base price of Rs. 100.

Solution:

Step 1: To find the price at the end of the second year we use the process of compounding

At the end of first year = $100\,(1 + 0.05)$

At the end of second year = $100\,(1 + 0.05)\,(1 + 0.1) = 115.5$

Step 2: To find the average inflation rate (f), we establish the equivalence relation.

At the end of second year,

Rs. $100\,(1 + f)^2 = 115.5$

Solving for f, $f = 7.47\%$

Money in one period of time t_1 can be brought to the same value as money in another period of time t_2, by using the relation.

$$\underbrace{\text{Dollars in period } t_2}_{\text{Future dollars}} = (\text{Inflation rate between } t_1 \text{ and } t_2) \times \underbrace{\text{Dollars in period } t_1}_{\text{Today dollars}}$$

8.3 Equivalence Calculation under Inflation

To introduce the factor in changes in purchasing power i.e. inflation, we use either

i) **Constant dollar analysis**

ii) **Actual dollar analysis**

i) **Actual dollar analysis (An):** (Out of pocket dollars paid at the time of purchasing goods and service.)

Actual dollars are estimates of future cash flows for year *n* that take into account any anticipated changes in amounts caused by inflationary or deflationary effects. Usually, these amounts are determined by applying an inflation rate to base-year dollar estimates.

ii) **Constant dollar analysis (A_n'):** (Dollars as if in some base year, used to adjust for the effects of inflation.)

Constant dollars represent constant purchasing power that is independent of the passage of time. In situations where inflationary effects were assumed when cash flows were estimated, the estimates obtained can be converted to constant dollars (base-year dollars) by adjustment with some readily accepted **general inflation rate.** We will assume that the base year is always time 0, unless we specify otherwise.

- **Conversion from constant to actual cash flow**

$$A_n = A_n'(1+f)^n$$

- **Conversion from actual to constant cash flow**

$$An' = \frac{An}{(1+f)^n}$$

8.3.1 Different interest rate

i) **Market interest rate (i) OR Inflation adjusted MARR :** The interest rate quoted by financial institutions that accounts for both earning purchasing power. This rate takes into account the combined effects of the earning value of capital (earning power) and any anticipated inflation or deflation (purchasing power). Virtually all interest rates stated by financial institutions for loans and savings accounts are market interest rates. Most firms use a market interest rate (also known as an **inflation-adjusted MARR**) in evaluating their investment projects. This is the interest rate that has been adjusted to take inflation into account.

ii) **Inflation-free interest (i') :** This rate is an estimate of the true earning power of money when the inflation effects have been removed. This is commonly known as the **real interest rate,** it can be computed if the market interest rate and the inflation rate are known. In the absence of inflation, the market interest rate is the same as the inflation-free interest rate

iii) **Inflation rate (f):** This is measure of the rate of change in the value or power of money.

Example: a) Convert the project's cash flows into the equivalent actual dollars. Inflation rate = 5%

Period	0	1	2	3	4
Net cash flow in constant dollar	– 450000	200000	220000	220000	230000

Solution:

Period	Constant dollar	Actual dollar
0	– 450000	$-450000(1+0.05)^0 = -450000$
1	200000	$200000 \times 1.05^1 = 210000$
2	220000	$220000 \times 1.05^2 = 242550$
3	220000	$220000 \times 1.05^3 = 254677.5$
4	230000	$230000 \times 1.05^4 = 279566.43$

b) The project is expected to generate the following cash flow in actual dollars.

Year	0	1	2	3	4
Actual dollar	– 750000	320000	357000	328000	290000

i) **What are the equivalent constant dollars? When f = 5%.**

ii) **Compute the present worth of these cash flows in constant dollars at i = 10%.**

Solution:

Year	Actual dollar	Cash flow in constant dollar, f = 5%	Cash flow in constant dollar f = 10%
0	– 750000	$-750000 \times 1.05^{-0} = -750000$	$-750000 \times 1.1^{-0} = -750000$
1	320000	$320000 \times 1.05^{-1} = 304761.9$	$304761.9 \times 1.1^{-1} = 277056.27$
2	357000	$357000 \times 1.05^{-2} = 323809.5$	$323809.5 \times 1.1^{-2} = 267611.17$
3	328000	$328000 \times 1.05^{-3} = 283338.73$	$283338.73 \times 1.1^{-3} = 212876.58$
4	290000	$290000 \times 1.05^{-4} = 238583.72$	$238583.72 \times 1.1^{-4} = 162955.88$
		Total PW =	170499.9

8.3.2 Adjusted-discount method

The two step process shown in previous example can be solve in one step by adjusted discount method, which performs deflation and discounting in one step. Mathematically, we can combine this two-step procedure into one with the formula

$$A'_n = \frac{\frac{An}{(1+f)^n}}{(1+i')^n} = \frac{An}{(1+f)^n(1+i')^n}$$

$$A'_n = \frac{An}{(1+f)^n(1+i')^n} \cdots\cdots(1)$$

Since the market interest rate (*i*) reflects both the earning power and the purchasing power, we have

$$A'_n = \frac{An}{(1+i)^n} \cdots\cdots\cdots\cdots(2)$$

The equivalent present-worth values in Equations (1) And (2) must be equal at year 0

$$\text{Therefore, } \frac{An}{\{(1+f)(1+i')\}^n} = \frac{An}{(1+i)^n}$$

This leads to the following relationship among f, i, i'

$$(1+i) = (1+f)(1+i')$$

$$\text{or, } 1+i = 1+i'+f\times i'+f$$

$$\therefore \quad i = i'+f+i'\times f$$

Example: Compute the equivalent PW of a previous example by adjusted discount method.

Solution: Given, i' = 0.1, f = 0.05

Where f = inflation rate, i' = inflation - free interest

Market interest rate (i) $= i' + i'\times f + f$

$= 0.1 + 0.1\times 0.05 + 0.05 = 0.155 = 15.5\%$

Year	Actual dollar	Constant dollar, i = 15.5%
0	-750000	$-750000 \times 1.155^{-0} = -750000$
1	320000	$320000 \times 1.155^{-1} = 277056.27$
2	357000	$357000 \times 1.155^{-2} = 267611.17$
3	328000	$328000 \times 1.155^{-3} = 212876.58$
4	290000	$290000 \times 1.155^{-4} = 162955.88$
	Total PW =	170499.9 (Same result as above example)

8.4 Impact of Inflation on Economic Evaluation

Impact of inflation on economic evaluation are listed below

1. Fixed income groups are hit hard while flexible income groups like businessman, speculators are benefited.
2. Resources are diverted from productive to unproductive sectors during production.
3. During inflation social evils like corruption, gambling, black marketing etc. flourish.
4. It creates uncertainties in the economy.
5. It raises the cost of holding money.
6. It creates socio-political unrests and people do not trust the government.
7. Inflation redistributes d income and wealth in the economy because rich becomes richer and poor becomes poorer.

A number of individual elements of project evaluations can be distorted by inflation.

These elements are summarized here.

- **Depreciation expense:** Depreciation expense is charged to taxable income in dollars of declining values; taxable income is overstated, resulting in higher taxes.
- **Salvage values:** Inflated salvage values combined with book values based on historical costs result in higher taxable gains.
- **Loan repayments:** Borrowers repay historical loan amounts with dollars of decreased purchasing power, reducing the cost of financing debt.
- **Working capital:** Known as a *working capital drain*, the cost of working capital requirement increases in an inflationary economy.
- **Rate of return and NPW:** Unless revenues are sufficiently increased to keep pace with inflation, tax effects and a working-capital drain result in a lower rate of return or a lower NPW.

There are two methods of evaluation

1. Estimate inflation effect by converting all cash flows to money units that have constant purchasing power i.e. constant (real) dollars
2. Estimate cash flows in the amount of money units actually exchanged at the time of each transaction. I.e. actual (future) dollars.

Old Question Solutions

1. **Define inflation. List out its effect, If the inflation rate is 5% per year and the market interest rate is 13% per year. What is the implied interest (inflation free) rate in inflationary economy? (2069 Bhadra)**

Solution: Inflation is the rate at which the general level of prices, goods and services is rising and subsequently purchasing power is falling.

Effects of inflation: Refer at 8.4

Given, Market rate (i) = 13%

Inflation rate (f) = 5%

Inflation free rate (i) = ?

We have, $i = i' \times f \times i' + f$

$0.13 = i' + 0.05i' + 0.05$

Solving $i' = 0.0761 \approx 7.61\%$

2. **A series of five constant dollar (or real dollar) income (beginning with $5000 at the end of the first year) are increasing at the rate of 7% per year for five years. Inflation free interest rate is 5% and inflation is 8%. Is it feasible investment if investment cost is $20000? (2069 Bhadra)**

Solution:

Inflation free interest rate (i') = 5%

Inflation (f) = 8%

Market rate (i) = $f + f \times i' + i' = 0.08 + 0.08 \times 0.05 + 0.05 = 0.134 = 13.4\%$

Investment = $20000

Income = $ 5000 at the end of 1st year and increasing 7% per year.

Year	Cash flow (constant dollar)
0	-20000
1	5000
2	5000 (1 + 0.07) = 5350
3	$5000 (1 + 0.07)^2 = 5724.5$
4	$5000 (1 + 0.07)^3 = 6125.22$
5	$5000 (1 + 0.07)^4 = 6553.98$

Since, the cash flows are in constant dollar we should are inflation free interest rate to calculate equivalent worth.

$PW = -20,000 + 5,000\ (1.05)^{-1} + 5,350(1.05)^{-2} + 5,724.5(1.05)^{-3} + 6,125.22(1.05)^{-4} + 6,553\ (1.05)^{-5} = 4725 > 0$

Therefore, Investment is economically feasible.

3. **Evaluate the PW of the following project:**

Initial investment = Rs. 1,00,000

Annual sales income = Rs. 40,000

Annual labour cost = Rs. 3,000

Annual material X = Rs. 2,000

Annual material Y = Rs. 1,000

Salvage value = 20% of initial investment

All are in the constant dollars.

Inflation rate for sales income, labour cost, material X, material Y and salvage value are 5%, 8%, 0%, 6% and 3% respectively for the project period. Take market interest rate = 20% project life is 4 years. (2070 Magh)

Solution: Market interest rate (i) = 20%

Note: when cash flow are in constant amount we should use inflation free rate (i') of interest.

Project life = 4 years

$i = i' + i' \times f + f$

$i'(1+f) = i - f$

$i' = \frac{i-f}{1+f} = \frac{0.2-f}{1+f}$

S.N.	Cash flow item	i'	Cash flow
1	Investment	-	-100000
2	Sales	(0.2 – 0.05) / (1 + 0.05) = 0.143	40000 (annually)
3	Labour	(0.2 – 0.08) / (1 + 0.08) = 0.11	3000 (annually)
4	Material X	(0.2 – 0) / (1 + 0) = 0.2	2000 (annually)
5	Material Y	(0.2 – 0.06) / (1 + 0.06) = 0.132	1000 (annually)
6	Salvage value	(0.2 – 0.03) / (1 + 0.03) = 0.165	0.2 × 100000 = 20000

$$PW = -1{,}00{,}000 + 40000 \times \frac{1.143^4 - 1}{0.143 \times 1.143^4} - 3{,}000 \times \frac{1.11^4 - 1}{0.11 \times 1.11^4} - 2000$$

$$\times \frac{1.2^4 - 1}{0.2 \times 1.2^4} - 1000 \times \frac{1.132^4 - 1}{0.132 \times 1.132^4} + \frac{20000}{1.165^4} = 9245.79 > 0, \text{ (OK)}$$

4. **The annual fuel cost required to operate a small solid waste treatment plant are projected to be Rs. 200000 without considering any future inflation. The best estimate indicates that the annual inflation free interest rate i' will be 6% and the general inflation rate, f, will be 5%, if the plant has the remaining useful life of four years. What is the present equivalent of its fuel costs? Use actual dollar analysis.** **(2070 Bhadra)**

Solution: $i' = 6\%$, $f = 5\%$

$$i = i' + i' \times f + f = 0.06 + 0.06 \times 0.05 + 0.05 = 0.113 = 11.3\%$$

Using actual dollars analysis.

EOY	Cash flow	Cash flow in actual dollar
1	200000	$200000\ (1 + 0.113)^1 = 222600$
2	200000	$200000\ (1 + 0.113)^2 = 247753.8$
3	200000	$200000\ (1 + 0.113)^3 = 275749.98$
4	200000	$200000\ (1 + 0.113)^4 = 306909.75$
	Total PW =	1053013.5

∴ Total present equivalent of fuel cost = Rs. 1053013.5

5. **First cost = \$ 80000, Sv = 10% of first cost. The general inflation rate = 5%**

EOY	1	2	3	4	5
Net cash flow in actual dollars	32000	35000	33000	29000	50000

Evaluate the PW by deflection method, if inflation free interest rate = 10% **(2071 Magh)**

Solution:

Salvage value at the end of 5th year = $0.1 \times 80000 = 8000$

Total cash flow at the end of 5th year = $50000 + 8000 = 58000$

Using deflation method,

EOY	Net cash flow in actual dollars	Constant dollars at 5% inflation rate	Constant dollar at 10% inflation free rate
0	-80000	$-80000\ (1 + 0.05)^{-0} = -80000$	$-80000\ (1 + 0.1)^{-0} = -80000$
1	32000	$32000\ (1 + 0.05)^{-1} = 30476.2$	$30476.2\ (1 + 0.1)^{-1} = 27705.6$
2	35000	$35000\ (1 + 0.05)^{-2} = 31746$	$31746 \times 1.1^{-2} = 26236.38$
3	33000	$33000\ (1 + 0.05)^{-3} = 28506.64$	$28506.64 \times 1.1^{-3} = 21417.46$
4	29000	$29000\ (1 + 0.05)^{-4} = 23858.37$	$23858.37 \times 1.1^{-4} = 16295.58$
5	58000	$58000\ (1 + 0.05)^{-5} = 45444.52$	$45444.52 \times 1.1^{-5} = 28217.5$
		Total PW	39872.52

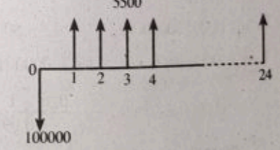
∴ PW = \$ 39872.52

6. **Define constant dollar amount and actual dollar amount. Suppose you borrowed Rs. 100000 from a bank to buy a bike and you have promised to pay Rs. 5500 per month for two years. What is the inflation free interest rate you are supposed to pay if average inflation rate is 0.75% per month?** **(2071 Bhadra)**

Solution: Theory part see at 8.3

Let i be the market interest rate per month.

Useful life = 2 years = 24 months.



Now, PW(i%) = 0

$$-100000 + 5500 \times \frac{[(1+i)^{24} - 1]}{i(1+i)^{24}} = 0$$

$i = 0.0235 = 2.35\%$

Average inflation rate (f) = 0.75%

Inflation free rate (i') = ?

$i = i' + f \times i' + f$

$i' = \frac{0.0235 - 0.0075}{1 + 0.0075} = 0.0158$

$\therefore$ Inflation free interest rate = 0.0158 = 1.58%

7. Choose the best project from the following alternatives

Project	Material X	Material Y
First cost	1500000	2000000
Life	7 years	7 years
Salvage value	200000	300000
Annual operation and maintenance cost	300000	250000

Assume an average inflation of 5% for the next five years and interest rate is 15%/years.
(2072 Ashwin)

Solution:

Inflation rate (r) = 5%

Market interest rate (i) = 15%

Inflation free interest (i') = ?

$i' = \frac{0.15 - 0.05}{1 + 0.05} = 0.095 = 9.5\%$ for next five years and

$i' = \frac{0.15 - 0}{1 + 0} = 15\%$ for 2 years

Assume constant dollar analysis

Note: Inflation is only for 5 years.

PW of X Machine = –15,00000+300000(P/A,9.5%,5)+300000(P/F,15%,6)
+ 300000 (P/F, 15%, 7) + 200000 (P/F, 15%, 7)

$= -15{,}00000 + 300000 * \frac{1.095^5 - 1}{1.095^5 * 0.095} + \frac{300000}{1.15^6}$

$+ \frac{300000}{1.15^7} + \frac{200000}{1.15^7} = -30420.56$

Also, PW of Y machine

$= -20{,}00000 + 250000 * \frac{1.095^6 - 1}{1.095^5 \times 0.095} + \frac{250000}{1.15^6} + \frac{25000}{1.15^7} + \frac{300000}{1.15^7}$

$= -725225.53$

Economically both projects aren't justified.

But NPW of X machine > NPW of Y machine

So, choose a machine X.

8. Define constant dollar and actual dollar amount. Suppose you borrowed Rs. 1, 20,000 from a bank to buy a bike and you have promised to pay Rs. 6,000 per month for two years. What is the inflation free interest rate you are supposed to pay if average inflation rate is 0.75% per month?
(2073 Bhadra)

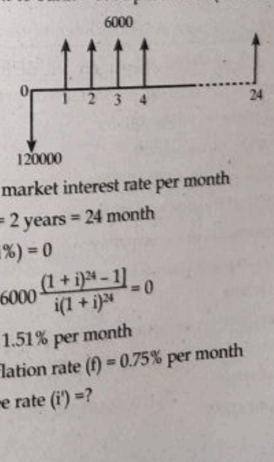
Solution:

Constant dollar amount : Constant dollar amount represents constant purchasing power independent of the passage of time. In situation where inflationary effects were assumed when cash flows were estimated these estimates can be converted to constant dollar by adjustment using some accepted general inflation rate. It is also known as on base year dollars.

Actual dollar amount : Actual dollars are estimates of future cash flow for year 'n' that take into account any anticipated changes in amount due to inflationary or deflationary effects. Usually these amounts are determined by applying on inflation rate to base year dollar estimates. It is also known as future dollars amount.

Given, Borrowed amount = 1,20,000

Paid amount to bank = 5500 per month (Like revenue for bank)



Let I be the market interest rate per month

Useful life = 2 years = 24 month

Now, PW (i%) = 0

$-120{,}000 + 6000 \frac{(1 + i)^{24} - 1]}{i(1 + i)^{24}} = 0$

i = 0.0151 = 1.51% per month

Average inflation rate (f) = 0.75% per month

Inflation free rate (i') =?

Now,

$$i = i' + f \times i' + f$$

$$i' = \frac{0.0151 - 0.0075}{1 + 0.0075} = 0.00754$$

∴ Inflation free interest rate = 0.00754 = 0.754%

9. **Define inflation. What are its causes? Find rate of inflation per year when price of a product has increased from Rs. 5, 00,000 to 6,30,000 over the period of 3 years.** (2073 Magh)

Solution:

Inflation: Inflation is the rate at which the general level of prices, good and services is rising and subsequently purchasing power is falling.

Causes of Inflation

- Inflation is a sustained rise in the general price level. Inflation can come from both the demand and the supply side of on economy.
- Inflation can arise from internal and external events.
- Some inflating pressures direct from the domestic economy.
- A rise I the rate of VAT would also be a cause of increased domestic inflation in the short term because it increases a firm's production costs.
- Fluctuation in the exchanges rate can also affect inflation.

Types of Inflation

1) **Demand - Pull inflation**

Demand pull inflation occurs when aggregate demand is growing at an unsustainable rate leading to increased pressure on scarce resources and a positive output gap.

Causes of demand – pull inflation

a) A depreciation of the exchange rate
b) Higher demand from a fiscal stimulus
c) Monetary stimulus to the economy
d) Fast growth in other countries.

2) **Cost - push inflation**

Cost - push inflation occurs when firms respond to rising costs by increasing prices in order to protect their profit margins.

a) Component costs
b) Rising labour costs

c) Expectations of inflation
d) Higher indirect taxes
e) A fall in the exchange rate
f) Monopoly employers/Profit - push inflation

Numerical Part

Rate of inflation (i) = ?

Price of a product has increased from Rs. 5,00,000 to Rs. 6,30,000

Time (N) = 3

$$\therefore F = P(1 + f)^N$$

$$6,30,000 = 5,00,000 (1 + i)^3$$

$$f = 0.08$$

$$\therefore f = 8\%$$

I.e. Rate of inflation = 8%

10. **Define inflation. Calculate IRR if MARR = 12% and inflation rate is 8 %.**

Year	0	1	2	3	4
Constant Dollar	- 6000	1500	2000	2500	3000

(2074 Bhadra)

Solution:

Inflation: It is the rate at which the general level of prices, goods and services is rising and subsequently purchasing power is falling.

Converting constant dollar in to actual dollar for f = 8 % = 0. 08

Year (n)	Constant dollar(A_n')	Actual dollar ($A_n = A_n'(1 + f)^n$
0	- 6000	$-600 \times (1 + 0.08)^0 = -6000$
1	1500	$1500 \times (1 + 0.08)^1 = 1620$
2	2000	$2000 \times (1 + 0.08)^2 = 2332.8$
3	2500	$2500 \times (1 + 0.08)^3 = 314928$
4	3000	$3000 \times (1 + 0.08)^4 = 4081.46$

To calculate IRR

PW = (i %) = 0

$$-6000 + \frac{1620}{(1 + i)} + \frac{23328}{(1 + i)^2} + \frac{31.928}{(1 + i)^3} + \frac{4081.46}{(1 + i)^4} = 0$$

By using calculator, (1 + i) = 1.255

∴ i = 0.255 = 25.5% > MARR. (OK)

11. What is inflation? List out the impact of inflation. Calculate the rate of inflation when CPI moves from 100 to 250 over three years. **(2075 Baishakh)**

Solution: *For definition Refer 8.1 and Refer 8.4 for impact of Inflation.*

Given that CPI moves from 100 to 250 over 3 years.

i.e. $250 = 100\,(1+f)^3$

On solving,

$\therefore$ Inflation rate (f) = 0.357 = 35.7 %

Solving for *f* yields, f = 6.49%.

i.e. Average inflation rates = 6.49 %

12. Calculate the equipment present worth of the project from the following cash flow. Assume inflation free interest rate at 5% and inflation as 10% respectively. **(2075 Bhadra)**

Eoy	Cash flow in Actual $
0	- 7,50,000
1	3,20,000
2	3,75,000
3	3,28,000
4	2,90,000
5	5,80,000

Solution:

Inflation free interest rate (i') = 5%

Inflation rate (f) = 10%

So, Market interest rate $(i) = i^1 + i^1 \times f + f$

= $0.10 + 0.10 \times 0.05 + 0.05$

= 0.155

= 15.5 %

Using market interest rate for actual dollar analysis.

Eoy	cash flow in Actual $	Discounting factor	Equivalent present worth
0	- 7,50,000	$(1.115)^{-0}$	-7,50,000
1	3,20,000	$(1.115)^{-1}$	2,77,056
2	3,75,000	$(1.115)^{-2}$	2,81,101
3	3,28,000	$(1.115)^{-3}$	2,12,877
4	2,90,000	$(1.115)^{-4}$	1,62,956
5	5,80,000	$(1.115)^{-5}$	2,82,175

$\therefore$ Equivalent PW = 7,50,000 + 2,77,056 + 2,81,101 + 2,12,877 + 1,62,956 + 2,82,175

= 4,66,165

13. Why inflation is important to engineers for economic analysis? Suppose that the year inflation rates for first year and second year are 5% and 8% respectively. Calculate the average inflation rate of two years if the base price is Rs 100. (2076 Bhadra)

Solution:

To find the price at the end of the second year, we use the process of compounding:

$$\text{Rs.}100\,(1 + 0.05)(1 + 0.08) = \text{Rs.}113.4$$

To find the average inflation rate *f*, we establish the following equivalence equation:

Rs. $100(F/P, f, 2)$ = Rs.113.4

Or, Rs. $100(1 + f)^2$ = Rs.113.4

Extra Solution

1. **Calculate rate of inflation for each year and average inflation over the years from the following information.**

Year	Costs
0	504
1	538.4
2	577
3	629.5

Solution: Inflation rate during year 1

$$f_1 = \frac{538.4 - 504}{504} = 6.83\%$$

Similarly, $$f_2 = \frac{577 - 538.4}{538.4} = 7.71\%$$

$$f_3 = \frac{629.5 - 577}{577} = 9.1\%$$

Average inflation rate (f) over the years

$$F = p(1+f)^N$$

$$629.5 = 504(1+f)^3$$

$$1+f = 1.0769$$

$$f = 7.69\%$$

∴ Average inflation rate (f) = 7.69%

2. **Consider a loan that can be arranged at a nominal interest rate of 10% compounded monthly. If the inflation rate is expected to be 5%. What will be the market interest rate?**

Solution:

If nominal of interest (i_m) and compounding period (m) is given

$$i_{eff} = \left(1 + \frac{i_m}{m}\right)^{eff} - 1$$

$$= \left(1 + \frac{0.1}{12}\right)^{12} - 1 = 0.1047 = 10.47\%$$

Inflation rate (f) = 5%

Now market rate (i) = $i_{eff} + f \times i_{eff} + f$

$$= 0.1047 + 0.05 \times 0.1047 + 0.05$$

$$= 0.1599$$

$$= 15.99\%$$

3. **The projected sales and net cash flows in constant dollar are as follows:**

Period	Unit Sales	Net cash flow in constant $
0	-	- 2,50,000
1	1,000	1,00,000
2	1,100	1,10,000
3	1,200	1,20,000
4	1,300	1,30,000
5	1,200	1,20,000

Calculate PW if inflation free rate = 12%

Solution:

When cash flows are in constant amount we should use inflation free rate of interest

PW (12%) = - 2,50,000 + 1,00,000 (P/A, 12%, 5)

+ 10,000 (P/G,12%, 4) + 20,000 (P/F, 12%, 5)

= $ 1,63,099

4. **Determine the best alternative from following condition by using constant and actual dollars analysis;**

Alternative 1: Development cost will be Rs. 300,000 the first year and will increase at a rate of 5% over the 5 - year period

Alternative 1: Development costs will be a constant Rs. 300,000 per year in terms of today's dollar over the 5 year period.

Assume inflation rate = 3.5% and MARR = 25%

Solution:

The costs for each of the two alternative

Year	Future Rs. started by A	Constant Rs. started by B
1	$300{,}000\,(1+0.05)^0 = 300{,}0000$	300,000
2	$300{,}000\,(1+0.05)^1 = 315000$	300,000
3	$300{,}000\,(1+0.05)^2 = 330750$	300,000
4	$300{,}000\,(1+0.05)^3 = 347287.5$	300,000
5	$300{,}000\,(1+0.05)^4 = 364651.87$	300,000

Using Constant Dollar Analysis

Converting the actual dollar of alternative 1 to the constant dollar by using deflation factor.

Year	Future Rs. started by A	Constant Rs. started by B
1	$300{,}000\,(1+0.035)^{-1} = 289855$	300,000
2	$315000\,(1+0.035)^{-2} = 294055.87$	300,000
3	$330750\,(1+0.035)^{-3} = 298317.55$	300,000
4	$347287.5\,(1+0.035)^{-4} = 302640.99$	300,000
5	$364651.87\,(1+0.035)^{-5} = 307027.08$	300,000

For constant dollar, calculate real interest rate i.e. inflation free interest rate (i')

$$i = i' + i' \times f + f$$

$$i' = \frac{(i-f)}{(i+f)},\ i = 25\% \text{ and } f = 3.5\%$$

$$\therefore\ i' = \frac{(0.25 - 0.035)}{(1+0.035)}$$

$$= 0.208$$

$$= 20.8\%$$

$$\text{PW of alternative 1} = \frac{289855}{(1+0.208)^1} + \frac{294055.87}{1.208^2} + \frac{298317.55}{1.208^3} + \frac{302640.99}{1.208^4} + \frac{307027.08}{1.208^5} = \text{Rs. } 872163$$

$$\text{PW of alternative 2} = 300{,}000\,(P/A, 20.8\%, 5)$$

$$= 300{,}000 \times \frac{1.208^5 - 1}{1.208^5 \times 0.248} = \text{Rs. } 881617.12$$

Using Actual Dollar Analysis

Year	Future Rs. started by A	Constant Rs. started by B
1	300,000	$300000\,(1+0.035)^1 + = 310500$
2	315,000	$300000\,(1.035)^2 + = 321367.5$
3	330750	$300000^{\circ}\,(1.035)^3 = 332615.36$
4	347287.5	$300000(1.035)^4 = 344256.9$
5	364651.87	$300000\,(1.035)^5 = 356305.89$

$$\text{PW of alt. 1} = \frac{300{,}000}{(1+0.25)^1} + \frac{315{,}000}{1.25^2} + \frac{330750}{1.25^3} + \frac{347287.5}{1.25^4} + \frac{364651.87}{1.25^5}$$

$$= \text{Rs. } 872682$$

$$\text{PW of alt. 2} = \frac{310500}{1.25} + \frac{321367.5}{1.25^2} + \frac{332615.36}{1.25^3} + \frac{344256.9}{1.25^4} + \frac{356305.89}{1.25^5}$$

$$= \text{Rs. } 882136.2$$

Using either Constant dollar or actual dollar, alternative 1 (minimum cost) should be choose.

□□□